Electricity

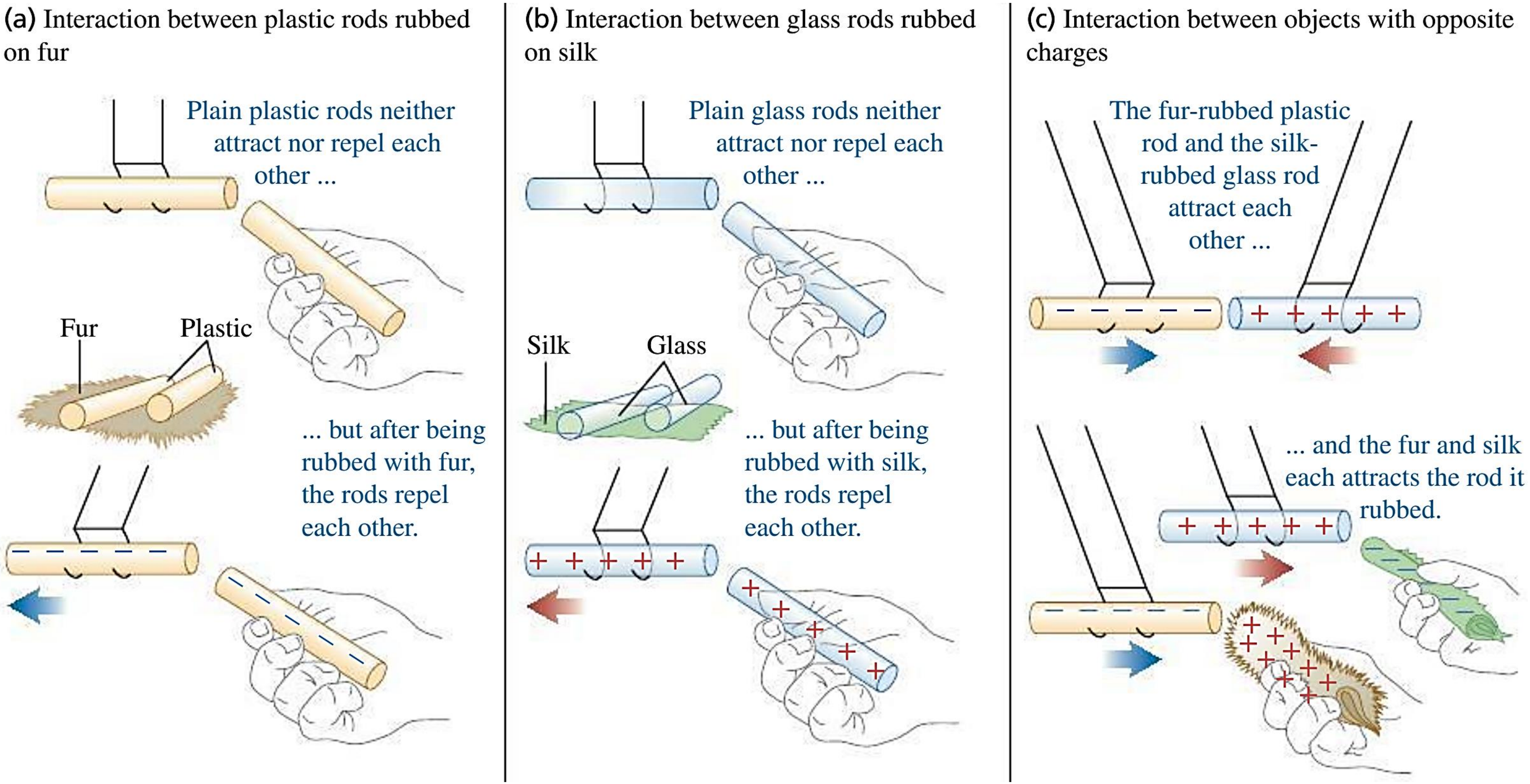
Electric charge

The ancient Greeks discovered as early as 600 b.c. that after they rubbed amber with wool, the amber could attract other objects. Today we say that the amber has acquired a net **electric charge,** or has become *charged*. The word "electric" is derived from the Greek word *elektron*, meaning amber. When you scuff your shoes across a nylon carpet, you become electrically charged, and you can charge a comb by passing it through dry hair.

Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics,** the interactions between electric charges that are at rest (or nearly so). After we charge both plastic rods in Figure a by rubbing them with the piece of fur, we find that the rods repel each other.

When we rub glass rods with silk, the glass rods also become charged and repel each other (Fig. b). But a charged plastic rod *attracts* a charged glass rod; furthermore, the plastic rod and the fur attract each other, and the glass rod and the silk attract each other (Fig. c).

These experiments and many others like them have shown that there are exactly two kinds of electric charge: the kind on the plastic rod rubbed with fur and the kind on the glass rod rubbed with silk. Benjamin Franklin (1706–1790) suggested calling these two kinds of charge *negative* and *positive*, respectively, and these names are still used. The plastic rod and the silk have negative charge; the glass rod and the fur have positive charge.

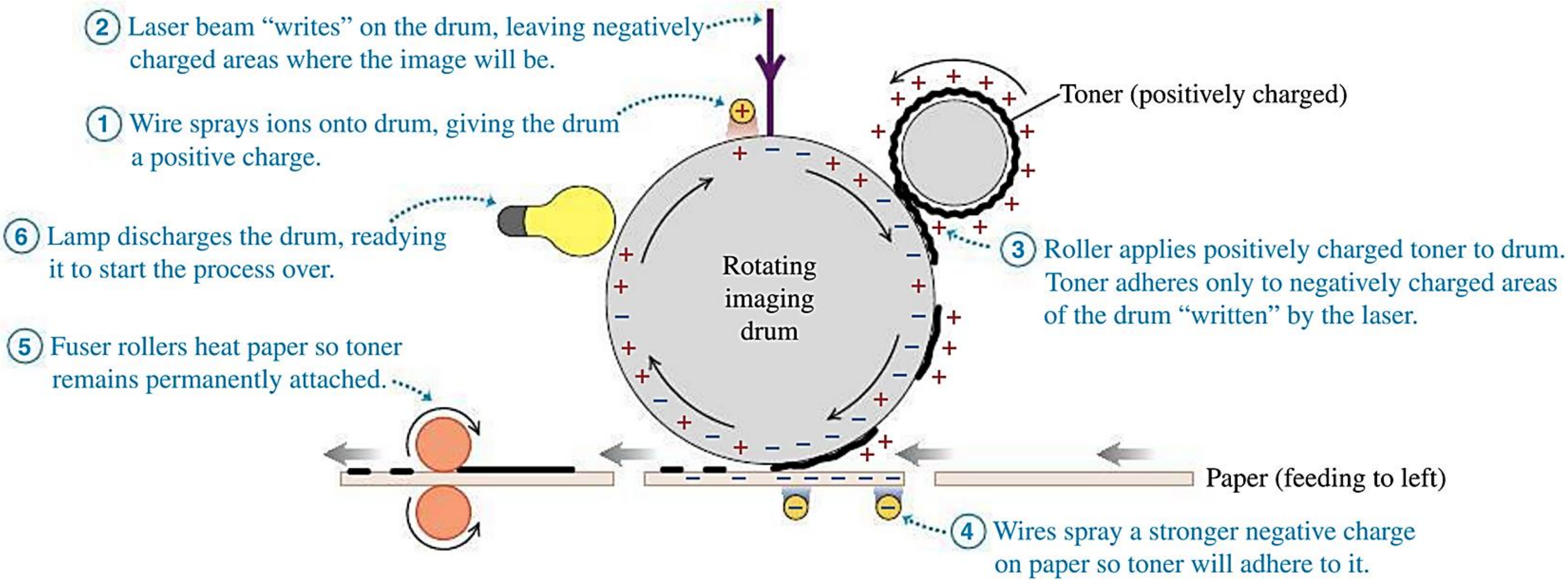


Experiments in electrostatics. (a) Negatively charged objects repel each other. (b) Positively charged objects repel each other. (c) Positively charged objects and negatively charged objects attract each other.

Two positive charges or two negative charges repel each other. A positive charge and a negative charge attract each other.

The attraction and repulsion of two charged objects are sometimes summarized as “Like charges repel, and opposite charges attract.” But “like charges” does *not* mean that the two charges are exactly identical, only that both charges have the same algebraic *sign* (both positive or both negative). “Opposite charges” means that both objects have an electric charge, and those charges have different signs (one positive and the other negative).

A laser printer (see the next Figure) utilizes the forces between charged bodies. The printer’s light-sensitive imaging drum is given a positive charge. As the drum rotates, a laser beam shines on selected areas of the drum, leaving those areas with a *negative* charge. Positively charged particles of toner adhere only to the areas of the drum “written” by the laser. When a piece of paper is placed in contact with the drum, the toner particles stick to the paper and form an image.



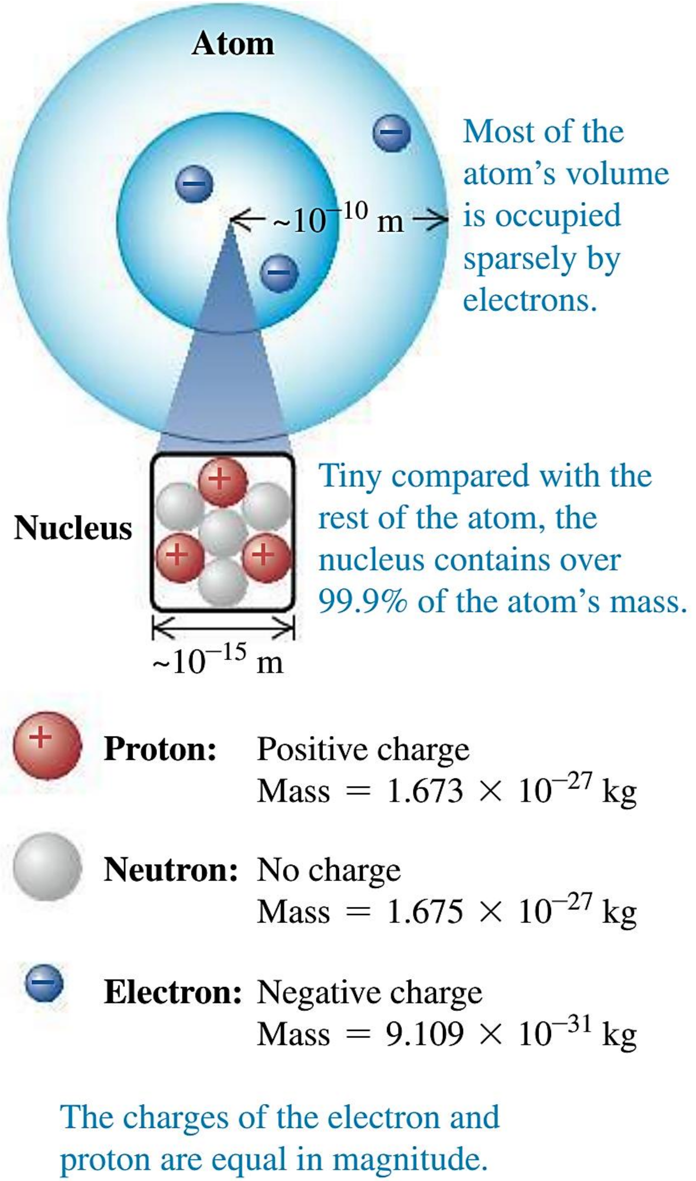
Schematic diagram of the operation of a laser printer.

Electric charge and the structure of matter

When you charge a rod by rubbing it with fur or silk as in Figure above, there is no visible change in the appearance of the rod. What, then, actually happens to the rod when you charge it? To answer this question, we must look more closely at the structure of atoms, the building blocks of ordinary matter.

The structure of atoms can be described in terms of three particles: the negatively charged **electron**, the positively charged **proton,** and the uncharged **neutron** (see the figure). The proton and neutron are combinations of other entities called *quarks*, which have charges of ± 1/3 and ± 2/3 times the electron charge. Isolated quarks have not been observed, and there are theoretical reasons to believe that it is impossible in principle to observe a quark in isolation.

The protons and neutrons in an atom make up a small, very dense core called the **nucleus,** with dimensions of the order of 10^{-15} m. Surrounding the nucleus are the electrons, extending out to distances of the order of 10^{-10} m from the nucleus. If an atom were a few kilometers across, its nucleus would be the size of a tennis ball. The negatively charged electrons are held within the atom by the attractive electric forces exerted on them by the positively charged nucleus. (The protons and neutrons are held within stable atomic nuclei by an attractive interaction, called the *strong nuclear force*, that overcomes the electric repulsion of the protons. The strong nuclear force has a short range, and its effects do not extend far beyond the nucleus.)



The structure of an atom. The particular atom depicted here is lithium (see the next Figure a).

The masses of the individual particles, to the precision that they are presently known, are

$$\text{Mass of electron} = m_e = 9.10938291(40) \times 10^{-31}\ \text{kg}$$

$$\text{Mass of proton} = m_p = 1.672621777(74) \times 10^{-27}\ \text{kg}$$

$$\text{Mass of neutron} = m_n = 1.674927351(74) \times 10^{-27}\ \text{kg}$$

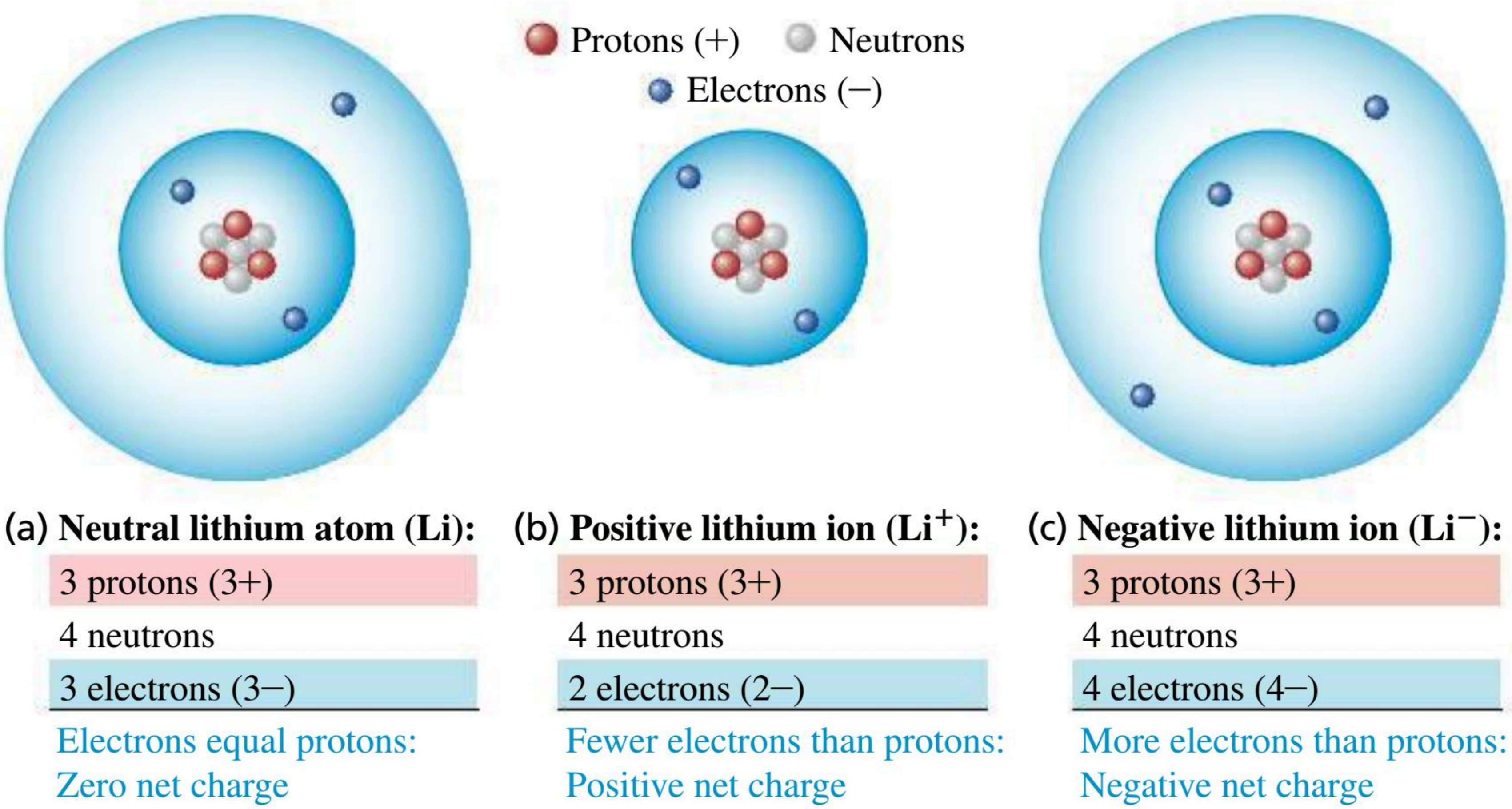
The numbers in parentheses are the uncertainties in the last two digits. Note that the masses of the proton and neutron are nearly equal and are roughly 2000 times the mass of the electron. Over 99.9% of the mass of any atom is concentrated in its nucleus.

The negative charge of the electron has (within experimental error) *exactly* the same magnitude as the positive charge of the proton. In a neutral atom the number of electrons equals the number of protons in the nucleus, and the net electric charge (the algebraic sum of all the charges) is exactly zero (see the next Figure a). The number of protons or electrons in a neutral atom of an element is called the **atomic number** of the element. If one or more electrons are removed from an atom, what remains is called a **positive ion** (Fig. b). A **negative ion** is an atom that has *gained* one or more electrons (Fig. c). This gain or loss of electrons is called **ionization.**

When the total number of protons in a macroscopic body equals the total number of electrons, the total charge is zero and the body as a whole is electrically neutral. To give a body an excess negative charge, we may either *add negative* charges to a neutral body or *remove positive* charges from that body. Similarly, we can create an excess positive charge by either *adding positive* charge or *removing negative* charge.

In most cases, negatively charged (and highly mobile) electrons are added or removed, and a "positively charged body" is one that has lost some of its normal complement of electrons. When we speak of the charge of a body, we always mean its *net* charge. The net charge is always a very small fraction (typically no more than 10^{-12}) of the total positive charge or negative charge in the body.

(a) A neutral atom has as many electrons as it does protons. (b) A positive ion has a deficit of electrons. (c) A negative ion has an excess of electrons. (The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus.)



Electric charge is conserved

Implicit in the foregoing discussion are two very important principles. First is the **principle of conservation of charge:**

The algebraic sum of all the electric charges in any closed system is constant.

If we rub together a plastic rod and a piece of fur, both initially uncharged, the rod acquires a negative charge (since it takes electrons from the fur) and the fur acquires a positive charge of the *same* magnitude (since it has lost as many electrons as the rod has gained). Hence the total electric charge on the two bodies together does not change. In any charging process, charge is not created or destroyed; it is merely *transferred* from one body to another.

Conservation of charge is thought to be a *universal* conservation law. No experimental evidence for any violation of this principle has ever been observed. Even in high-energy interactions in which particles are created and destroyed, such as the creation of electron–positron pairs, the total charge of any closed system is exactly constant.

The second important principle is:

The magnitude of charge of the electron or proton is a natural unit of charge.

Every observable amount of electric charge is always an integer multiple of this basic unit. We say that charge is *quantized.* A familiar example of quantization is money. When you pay cash for an item in a store, you have to do it in one-cent increments.

Cash can’t be divided into amounts smaller than one cent, and electric charge can’t be divided into amounts smaller than the charge of one electron or proton. (The quark charges, ± 1/3 and ± 2/3 of the electron charge, are probably not observable as isolated charges.) Thus the charge on any macroscopic body is always zero or an integer multiple (negative or positive) of the electron charge.

Understanding the electric nature of matter gives us insight into many aspects of the physical world (see the Figure). The chemical bonds that hold atoms together to form molecules are due to electric interactions between the atoms. They include the strong ionic bonds that hold sodium and chlorine atoms together to make table salt and the relatively weak bonds between the strands of DNA that record your body’s genetic code. When you stand, the normal force exerted on you by the floor arises from electric forces between charged particles in the atoms of your shoes and the atoms of the floor. The tension force in a stretched string and the adhesive force of glue are likewise due to electric interactions of atoms.

Most of the forces on this water skier are electric. Electric interactions between adjacent molecules give rise to the force of the water on the ski, the tension in the tow rope, and the resistance of the air on the skier’s body. Electric interactions also hold the atoms of the skier’s body together. Only one wholly nonelectric force acts on the skier: the force of gravity.

Coulomb's law

Charles Augustin de Coulomb (1736–1806) studied the interaction forces of charged particles in detail in 1784. He used a torsion balance (see the Figure a) similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction. For **point charges,** charged bodies that are very small in comparison with the distance r between them, Coulomb found that the electric force is proportional to $1/r^2$. That is, when the distance r doubles, the force decreases to one-quarter of its initial value; when the distance is halved, the force increases to four times its initial value.

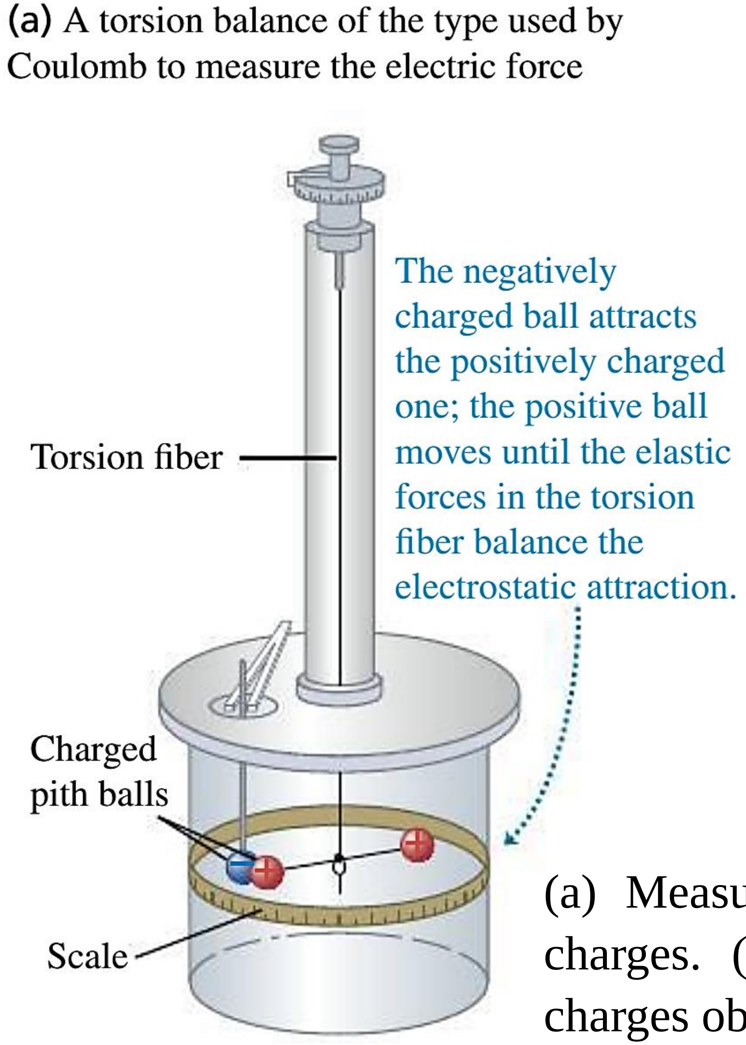
The electric force between two point charges also depends on the quantity of charge on each body, which we will denote by q or Q. To explore this dependence, Coulomb divided a charge into two equal parts by placing a small charged spherical conductor into contact with an identical but uncharged sphere; by symmetry, the charge is shared equally between the two spheres. (Note the essential role of the principle of conservation of charge in this procedure.) Thus he could obtain one-half, one-quarter, and so on, of any initial charge. He found that the forces that two point charges q_1 and q_2 exert on each other are proportional to each charge and therefore are proportional to the *product* q_1q_2 of the two charges.

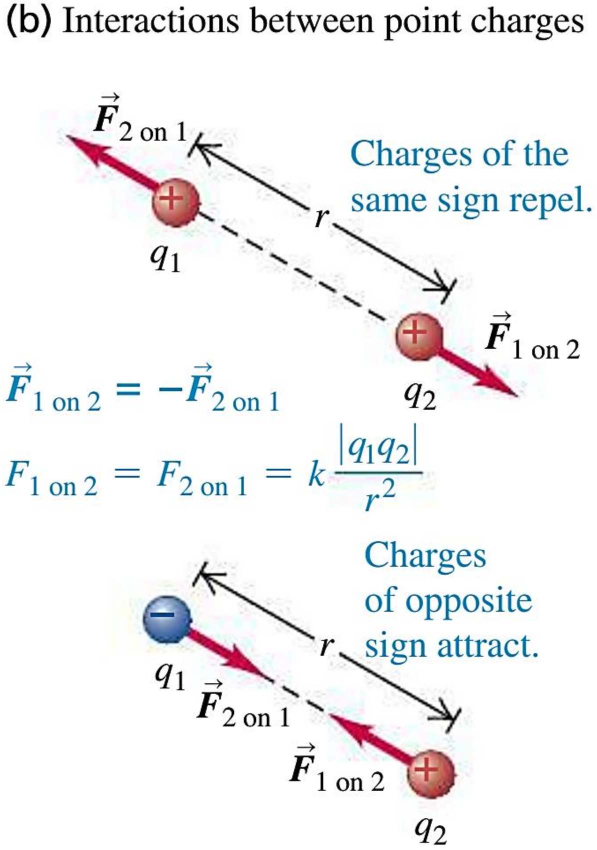
Thus Coulomb established what we now call **Coulomb's law:**

The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

In mathematical terms, the magnitude F of the force that each of two point charges q_1 and q_2 a distance r apart exerts on the other can be expressed as

$$F = k\frac{|q_1 q_2|}{r^2}$$





(a) Measuring the electric force between point charges. (b) The electric forces between point charges obey Newton's third law: $\boldsymbol{F}_{1\text{ on }2} = -\boldsymbol{F}_{2\text{ on }1}$.

where k is a proportionality constant whose numerical value depends on the system of units used. The absolute value bars are used in the Equation of Coulomb's law because the charges q_1 and q_2 can be either positive or negative, while the force magnitude F is always positive.

The directions of the forces the two charges exert on each other are always along the line joining them. When the charges q_1 and q_2 have the same sign, either both positive or both negative, the forces are repulsive; when the charges have opposite signs, the forces are attractive (Fig. b).

The two forces obey Newton's third law; they are always equal in magnitude and opposite in direction, even when the charges are not equal in magnitude.

The proportionality of the electric force to $1/r^2$ has been verified with great precision. There is no reason to suspect that the exponent is different from precisely 2. Thus the form of Coulomb's law Equation is the same as that of the law of gravitation. But electric and gravitational interactions are two distinct classes of phenomena. Electric interactions depend on electric charges and can be either attractive or repulsive, while gravitational interactions depend on mass and are always attractive (because there is no such thing as negative mass).

Fundamental electric constants

The value of the proportionality constant k in Coulomb's law depends on the system of units used. In our study of electricity and magnetism we will use SI units exclusively. The SI electric units include most of the familiar units such as the volt, the ampere, the ohm, and the watt. (There is *no* British system of electric units.) The SI unit of electric charge is called one **coulomb** (1 C). In SI units the constant k in the Equation of Coulomb's law is

$$k = 8.987551787 \times 10^9\ \mathrm{N \cdot m^2/C^2} \cong 8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}$$

The value of k is known to such a large number of significant figures because this value is closely related to the speed of light in vacuum. (We will show this later when we study electromagnetic radiation.) As you all know, this speed is *defined* to be exactly $c = 2.99792458 \times 10^8$ m/s. The numerical value of k is defined in terms of c to be precisely

$$k = (10^{-7}\ \mathrm{N \cdot s^2/C^2})c^2$$

You should check this expression to confirm that k has the right units.

In principle we can measure the electric force F between two equal charges q at a measured distance r and use Coulomb's law to determine the charge. Thus we could regard the value of k as an operational definition of the coulomb. For reasons of experimental precision it is better to define the coulomb instead in terms of a unit of electric *current* (charge per unit time), the *ampere*, equal to 1 coulomb per second. We will return to this definition later.

In SI units we usually write the constant k as $1/4\varepsilon_0$, where ε_0 ("epsilon-nought" or "epsilon-zero") is called the **electric constant.** This shorthand simplifies many formulas that we will encounter in later chapters. From now on, we will usually write Coulomb's law as

Coulomb's law:
Magnitude of electric force between two point charges

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

Values of the two charges

Electric constant

Distance between the two charges

The constants in this Equation are approximately

$$\epsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2} \quad \text{and} \quad \frac{1}{4\pi\epsilon_0} = k = 8.988 \times 10^{9}\ \mathrm{N\cdot m^2/C^2}$$

In problems we will often use the approximate value

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^{9}\ \mathrm{N\cdot m^2/C^2}$$

As we mentioned above, the most fundamental unit of charge is the magnitude of the charge of an electron or a proton, which is denoted by e. From the 2019 redefinition of SI base units, which took effect on 20 May 2019, its value is *exactly*

$$1.602176634 \times 10^{-19}\ \mathrm{C}.$$

One coulomb represents the negative of the total charge of about 6×10^{18} electrons. For comparison, a copper cube 1 cm on a side contains about 2.4×10^{24} electrons. About 10^{19} electrons pass through the glowing filament of a flashlight bulb every second.

In electrostatics problems (problems that involve charges at rest), it's very unusual to encounter charges as large as 1 coulomb. Two 1-C charges separated by 1 m would exert forces on each other of magnitude 9×10^{9} N (about 1 million tons)! The total charge of all the electrons in a copper one-cent coin is even greater, about 1.4×10^{5} C, which shows that we can't disturb electric neutrality very much without using enormous forces. More typical values of charge range from about a microcoulomb ($1\ \mu\mathrm{C} = 10^{-6}$ C) to about a nanocoulomb ($1\ \mathrm{nC} = 10^{-9}$ C).

Superposition of forces

Coulomb's law as we have stated it describes only the interaction of two *point* charges. Experiments show that when two charges exert forces simultaneously on a third charge, the total force acting on that charge is the *vector sum* of the forces that the two charges would exert individually. This important property, called the **principle of superposition of forces,** holds for any number of charges.

By using this principle, we can apply Coulomb’s law to *any* collection of charges. Two of the examples at the end of this section use the superposition principle.

Strictly speaking, Coulomb’s law as we have stated it should be used only for point charges *in a vacuum*. If matter is present in the space between the charges, the net force acting on each charge is altered because charges are induced in the molecules of the intervening material. We will describe this effect later. As a practical matter, though, we can use Coulomb’s law unaltered for point charges in air. At normal atmospheric pressure, the presence of air changes the electric force from its vacuum value by only about one part in 2000.

Electric field and electric forces

When two electrically charged particles in empty space interact, how does each one know the other is there? We can begin to answer this question, and at the same time reformulate Coulomb’s law in a very useful way, by using the concept of *electric field*. electric field

To introduce this concept, let’s look at the mutual repulsion of two positively charged bodies *A* and *B* (see the Figure a). Suppose *B* has charge q_0, and let $\boldsymbol{F}_0$ be the electric force of *A* on *B*. One way to think about this force is as an “actionat-a-distance” force – that is, as a force that acts across empty space without needing physical contact between *A* and *B*. (Gravity can also be thought of as an “action-at-a-distance” force.) But a more fruitful way to visualize the repulsion between *A* and *B* is as a two-stage process. We first envision that body *A*, as a result of the charge that it carries, somehow *modifies the properties of the space around it*. Then body *B*, as a result of the charge that *it* carries, senses how space has been modified at its position. The response of body *B* is to experience the force $\boldsymbol{F}_0$.

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.

To find out experimentally whether there is an electric field at a particular point, we place a small charged body, which we call a **test charge,** at the point (Fig. c). If the test charge experiences an electric force, then there is an electric field at that point. This field is produced by charges other than q_0.

Force is a vector quantity, so electric field is also a vector quantity. (Note the use of vector signs as well as boldface letters and plus, minus, and equals signs in the following discussion.) We define the *electric field* $\boldsymbol{E}$ at a point as the electric force $\boldsymbol{F}_0$ experienced by a test charge q_0 at the point, divided by the charge q_0. That is, the electric field at a certain point is equal to the *electric force per unit charge* experienced by a charge at that point:

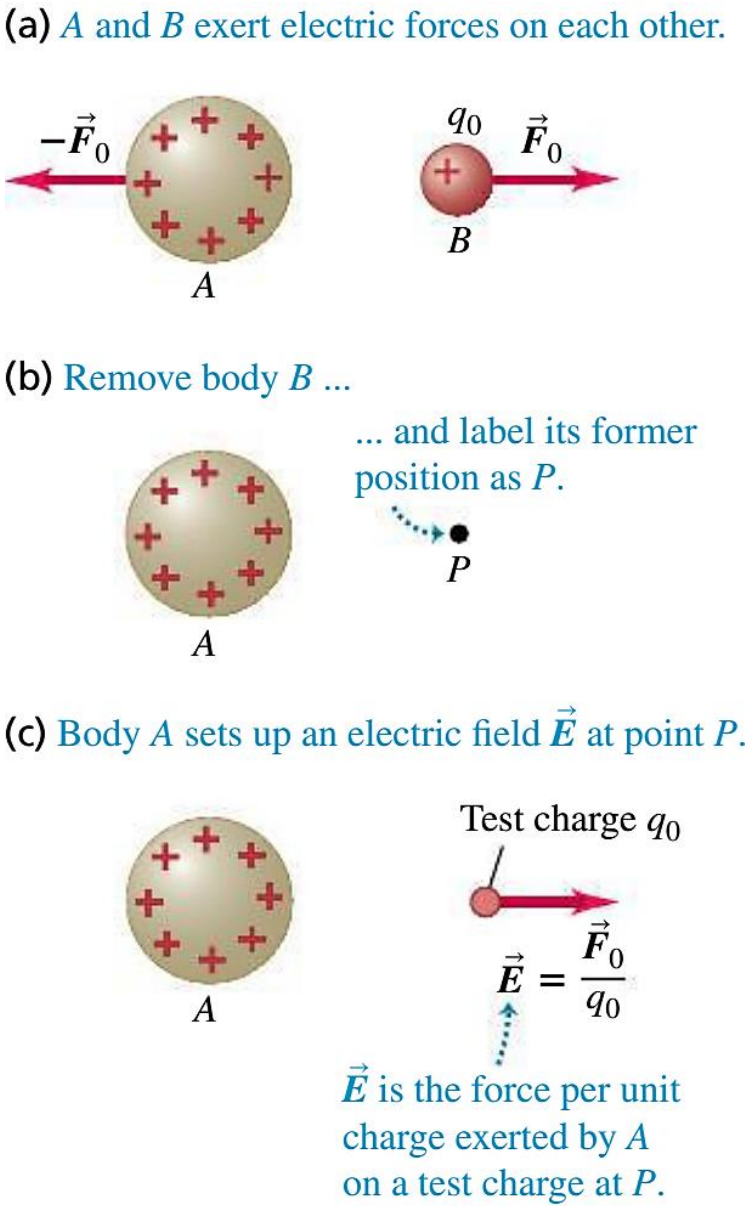
Electric field = electric force per unit charge

$$\vec{E} = \frac{\vec{F}_0}{q_0}$$

Electric force on a test charge q_0 due to other charges

Value of test charge

In SI units, in which the unit of force is 1 N and the unit of charge is 1 C, the unit of electric-field magnitude is 1 newton per coulomb (1 N/C).



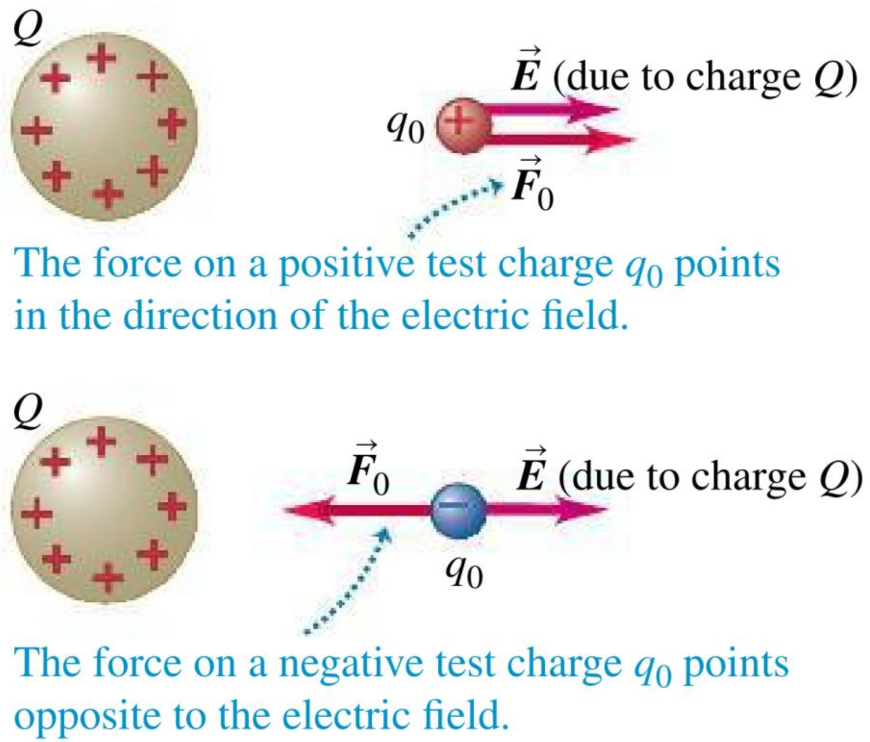
A charged body creates an electric field in the space around it.

If the field $\boldsymbol{E}$ at a certain point is known, rearranging the Equation gives the force $\boldsymbol{F}_0$ experienced by a point charge q_0 placed at that point. This force is just equal to the electric field $\boldsymbol{E}$ produced at that point by charges other than q_0, multiplied by the charge q_0:

$$\vec{F}_0 = q_0\vec{E} \qquad \text{(force exerted on a point charge } q_0 \text{ by an electric field } \vec{E}\text{)}$$

The charge q_0 can be either positive or negative. If q_0 is *positive*, the force $\boldsymbol{F}_0$ experienced by the charge is in the same direction as $\boldsymbol{E}$; if q_0 is *negative*, $\boldsymbol{F}_0$ and $\boldsymbol{E}$ are in opposite directions (see the Figure).

Caution! $\boldsymbol{F}_0 = q_0\boldsymbol{E}$ is for *point* test charges only The electric force experienced by a test charge q_0 can vary from point to point, so the electric field can also be different at different points. For this reason, use the Equation to find the electric force on a *point* charge only. If a charged body is large enough in size, the electric field $\boldsymbol{E}$ may be noticeably different in magnitude and direction at different points on the body, and calculating the net electric force on it can be complicated.



The force $\boldsymbol{F}_0 = q_0\boldsymbol{E}$ exerted on a point charge q_0 placed in an electric field $\boldsymbol{E}$

While the electric field concept may be new to you, the basic idea – that one body sets up a field in the space around it and a second body responds to that field – is one that you've actually used before. Compare the last Equation to the familiar expression for the gravitational force $\boldsymbol{F}_g$ that the earth exerts on a mass m_0:

$$\vec{\boldsymbol{F}}_g = m_0 \vec{\boldsymbol{g}}$$

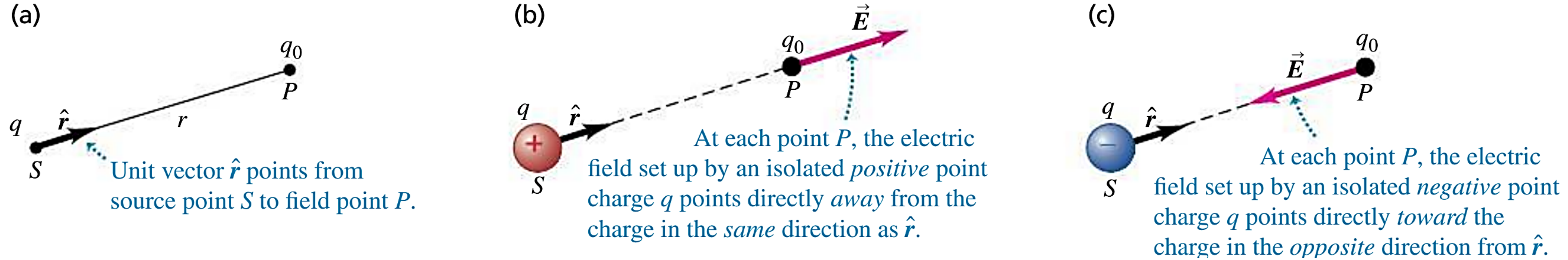
In this expression, $\boldsymbol{g}$ is the acceleration due to gravity. If we divide both sides of this Equation by the mass m_0, we obtain

$$\vec{\boldsymbol{g}} = \frac{\vec{\boldsymbol{F}}_g}{m_0}$$

Thus $\boldsymbol{g}$ can be regarded as the gravitational force per unit mass. By analogy to the electric field $\boldsymbol{E}$, we can interpret $\boldsymbol{g}$ as the *gravitational field*. Thus we treat the gravitational interaction between the earth and the mass *m*0 as a two-stage process: The earth sets up a gravitational field $\boldsymbol{g}$ in the space around it, and this gravitational field exerts a force on the mass m_0 (which we can regard as a *test mass*). The gravitational field $\boldsymbol{g}$, or gravitational force per unit mass, is a useful concept because it does not depend on the mass of the body on which the gravitational force is exerted; likewise, the electric field $\boldsymbol{E}$, or electric force per unit charge, is useful because it does not depend on the charge of the body on which the electric force is exerted.

Electric field of a point charge

If the source distribution is a point charge q, it is easy to find the electric field that it produces. We call the location of the charge the **source point,** and we call the point P where we are determining the field the **field point.** It is also useful to introduce a *unit vector* $\hat{r}$ that points along the line from source point to field point (see the Figure a).



The electric field $\boldsymbol{E}$ produced at point P by an isolated point charge q at S. Note that in both (b) and (c), $\boldsymbol{E}$ is *produced* by q but *acts* on the charge q_0 at point P.

This unit vector is equal to the displacement vector $\vec{r}$ from the source point to the field point, divided by the distance $r = |\vec{r}|$ between these two points; that is, $\hat{r} = \dfrac{\vec{r}}{r}$. If we place a small test charge q_0 at the field point P, at a distance r from the source point, the magnitude F_0 of the force is given by Coulomb's law:

$$F_0 = \frac{1}{4\pi\epsilon_0}\frac{|qq_0|}{r^2}$$

So the magnitude E of the electric field at P is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2} \qquad \text{(magnitude of electric field of a point charge)}$$

Using the unit vector $\hat{\boldsymbol{r}}$, we can write a *vector* equation that gives both the magnitude and direction of the electric field $\vec{\boldsymbol{E}}$:

Electric field due to a point charge

Value of point charge

Unit vector from point charge toward where field is measured

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

Electric constant

Distance from point charge to where field is measured

By definition, the electric field of a point charge always points *away from* a positive charge (that is, in the same direction as $\hat{r}$; see Fig. b) but *toward* a negative charge (that is, in the direction opposite $\hat{r}$; see Fig. c).

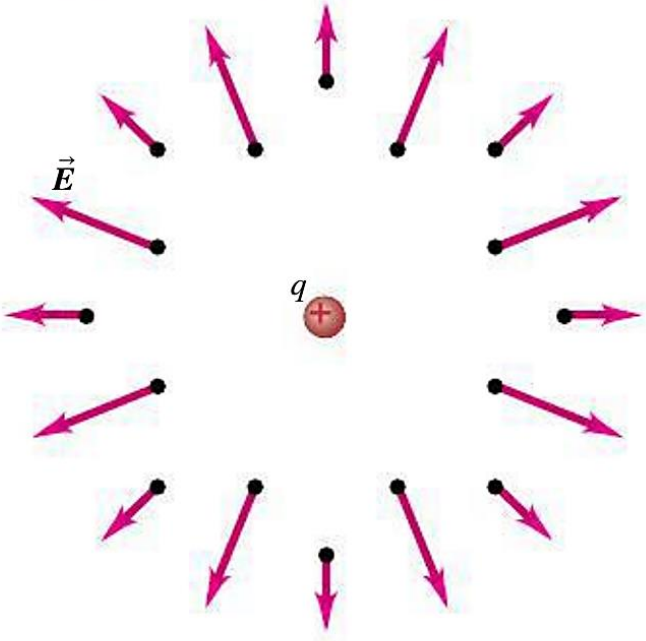
We have emphasized calculating the electric field ***E*** at a certain point. But since ***E*** can vary from point to point, it is not a single vector quantity but rather an *infinite* set of vector quantities, one associated with each point in space. This is an example of a **vector field**. The next Figure shows a number of the field vectors produced by a positive or negative point charge. If we use a rectangular (x, y, z) coordinate system, each component of ***E*** at any point is in general a function of the coordinates (x, y, z) of the point. We can represent the functions as $E_x(x, y, z)$, $E_y(x, y, z)$, and $E_z(x, y, z)$. Another example of a vector field is the velocity ***v*** of wind currents; the magnitude and direction of ***v***, and hence its vector components, vary from point to point in the atmosphere.

In some situations the magnitude and direction of the field (and hence its vector components) have the same values everywhere throughout a certain region; we then say that the field is *uniform* in this region. An important example of this is the electric field inside a *conductor*. If there is an electric field within a conductor, the field exerts a force on every charge in the conductor, giving the free charges a net motion.

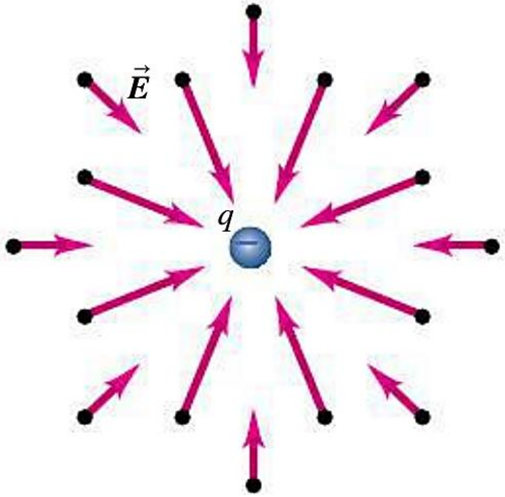
By definition an electrostatic situation is one in which the charges have *no* net motion. We conclude that *in electrostatics the electric field at every point within the material of a conductor must be zero.* (Note that we are not saying that the field is necessarily zero in a *hole* inside a conductor.)

In summary, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in the field. Our analysis often has two corresponding steps: first, calculating the field caused by a source charge distribution; second, looking at the effect of the field in terms of force and motion. The second step often involves Newton's laws as well as the principles of electric interactions. In the next section we show how to calculate fields caused by various source distributions, but first here are three examples of calculating the field due to a point charge and of finding the force on a charge due to a given field ***E***.

(a) The field produced by a positive point charge points *away from* the charge.



(b) The field produced by a negative point charge points *toward* the charge.



A point charge q produces an electric field ***E*** at *all* points in space. The field strength decreases with increasing distance.

The superposition of electric fields

To find the field caused by a charge distribution, we imagine the distribution to be made up of many point charges q_1, q_2, q_3, …. (This is actually quite a realistic description, since we have seen that charge is carried by electrons and protons that are so small as to be almost pointlike.) At any given point P, each point charge produces its own electric field $\boldsymbol{E}_1$, $\boldsymbol{E}_2$, $\boldsymbol{E}_3$, …, so a test charge q_0 placed at P experiences a force $\boldsymbol{F}_1 = q_0\boldsymbol{E}_1$ from charge q_1, a force $\boldsymbol{F}_2 = q_0\boldsymbol{E}_2$ from charge q_2, and so on. From the principle of superposition of forces discussed above, the *total* force $\boldsymbol{F}_0$ that the charge distribution exerts on q_0 is the vector sum of these individual forces:

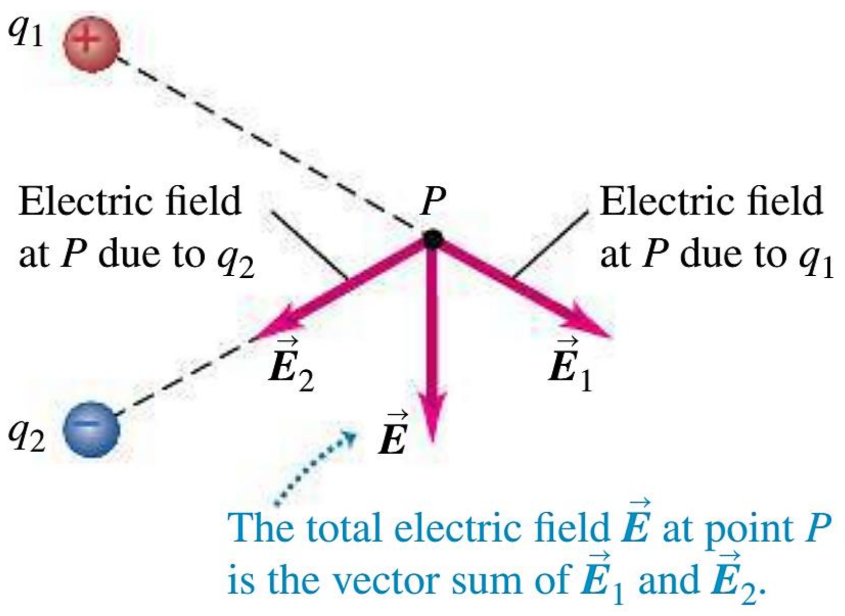
$$\vec{\boldsymbol{F}}_0 = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2 + \vec{\boldsymbol{F}}_3 + \cdots = q_0\vec{\boldsymbol{E}}_1 + q_0\vec{\boldsymbol{E}}_2 + q_0\vec{\boldsymbol{E}}_3 + \cdots$$

The combined effect of all the charges in the distribution is described by the *total* electric field $\boldsymbol{E}$ at point P. From the definition of electric field, this is

$$\vec{\boldsymbol{E}} = \frac{\vec{\boldsymbol{F}}_0}{q_0} = \vec{\boldsymbol{E}}_1 + \vec{\boldsymbol{E}}_2 + \vec{\boldsymbol{E}}_3 + \cdots$$

The total electric field at P is the vector sum of the fields at P due to each point charge in the charge distribution (see the Figure). This statement is the **principle of superposition of electric fields.**

When charge is distributed along a line, over a surface, or through a volume, a few additional terms are useful.



Illustrating the principle of superposition of electric fields.

For a line charge distribution (such as a long, thin, charged plastic rod), we use λ (the Greek letter lambda) to represent the **linear charge density** (charge per unit length, measured in C/m). When charge is distributed over a surface (such as the surface of the imaging drum of a laser printer), we use σ (sigma) to represent the **surface charge density** (charge per unit area, measured in C/m2). And when charge is distributed through a volume, we use ρ (rho) to represent the **volume charge density** (charge per unit volume, C/m3).

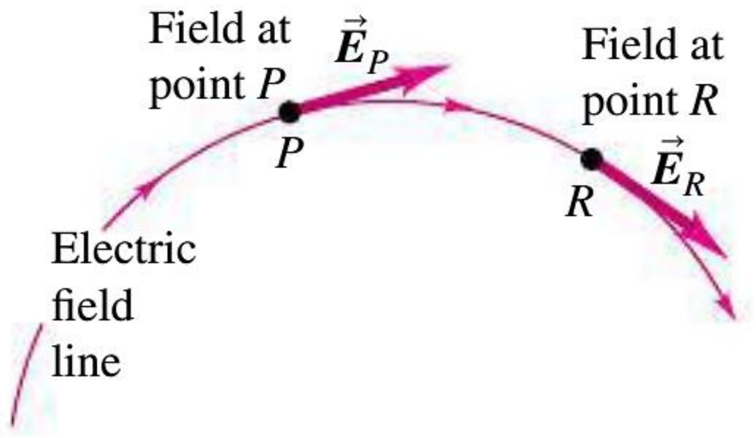
When students were given a problem involving electric force and electric field, more than 28% gave an incorrect response. Common errors:

● Forgetting that the electric field $\boldsymbol{E}$ experienced by a point charge does not depend on the value of that point charge. The value of $\boldsymbol{E}$ is determined by the charges that produce the field, not the charge that experiences it.

● Forgetting that $\boldsymbol{E}$ is a vector. If the field $\boldsymbol{E}$ at point P is due to two or more point charges, $\boldsymbol{E}$ is the vector sum of the fields due to the individual charges. In general, this is not the sum of the magnitudes of these fields.

Electric field lines

The concept of an electric field can be a little elusive because you can't see an electric field directly. Electric field *lines* can be a big help for visualizing electric fields and making them seem more real. An **electric field line** is an imaginary line or curve drawn through a region of space so that its tangent at any point is in the direction of the electric-field vector at that point. The Figure shows the basic idea. (We used a similar concept in our discussion of fluid flow in previous Section of our lectures. A *streamline* is a line or curve whose tangent at any point is in the direction of the velocity of the fluid at that point. However, the similarity between electric field lines and fluid streamlines is a mathematical one only; there is nothing "flowing" in an electric field.)

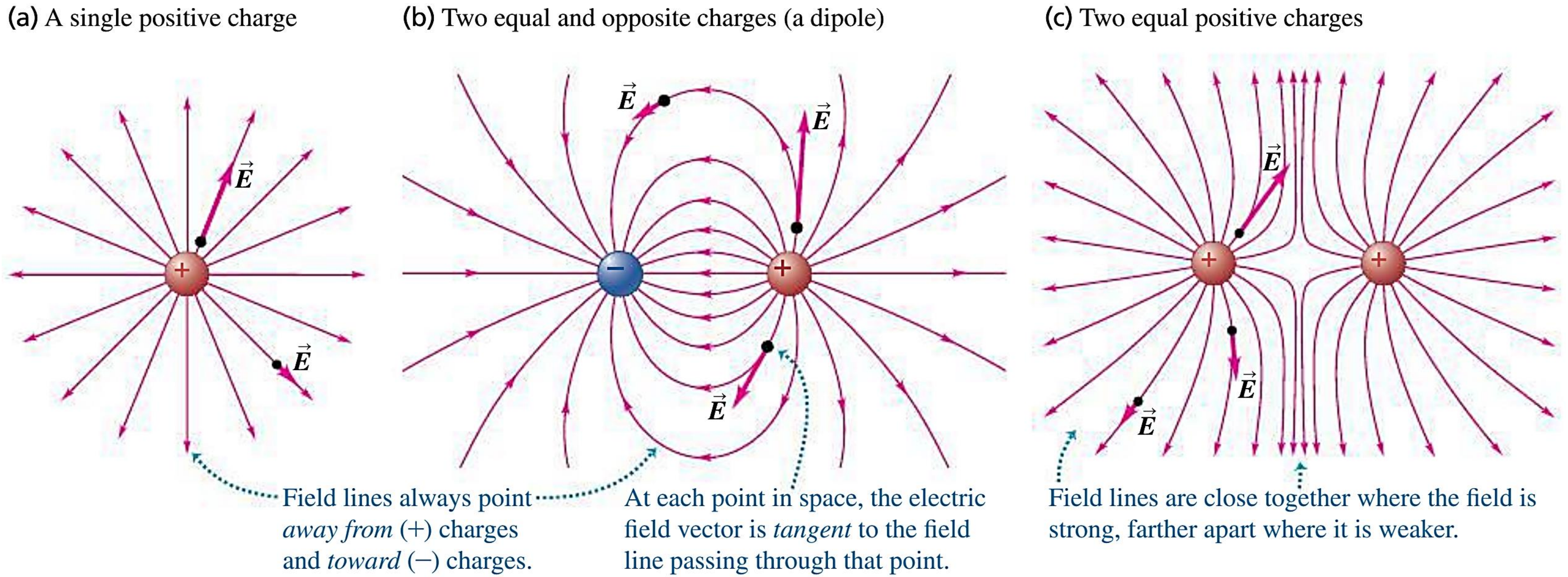


The direction of the electric field at any point is tangent to the field line through that point.

The English scientist Michael Faraday (1791–1867) first introduced the concept of field lines. He called them "lines of force," but the term "field lines" is preferable.

Electric field lines show the direction of $\boldsymbol{E}$ at each point, and their spacing gives a general idea of the *magnitude* of $\boldsymbol{E}$ at each point. Where $\boldsymbol{E}$ is strong, we draw lines close together; where $\boldsymbol{E}$ is weaker, they are farther apart. At any particular point, the electric field has a unique direction, so only one field line can pass through each point of the field. In other words, *field lines never intersect.*

The Figure shows some of the electric field lines in a plane containing (a) a single positive charge; (b) two equal-magnitude charges, one positive and one negative (a dipole); and (c) two equal positive charges. Such diagrams are called *field maps;* they are cross sections of the actual three-dimensional patterns. The direction of the total electric field at every point in each diagram is along the tangent to the electric field line passing through the point. Arrowheads indicate the direction of the ***E***-field vector along each field line.



Electric field lines for three different charge distributions. In general, the magnitude of ***E*** is different at different points along a given field line.

The actual field vectors have been drawn at several points in each pattern. Notice that in general, the magnitude of the electric field is different at different points on a given field line; a field line is *not* a curve of constant electric-field magnitude!

The Figure shows that field lines are directed *away* from positive charges (since close to a positive point charge, ***E*** points away from the charge) and *toward* negative charges (since close to a negative point charge, ***E*** points toward the charge). In regions where the field magnitude is large, such as between the positive and negative charges in Fig. b, the field lines are drawn close together. In regions where the field magnitude is small, such as between the two positive charges in Fig. c, the lines are widely separated. In a *uniform* field, the field lines are straight, parallel, and uniformly spaced.

Electric potential

This section is about energy associated with electrical interactions. Every time you turn on a light, use a mobile phone, or make toast in a toaster, you are using electrical energy, an indispensable ingredient of our technological society. The concepts of *work* and *energy* was introduced in the context of mechanics; now we'll combine these concepts with what we've learned about electric charge, electric forces, and electric fields. Just as we found for many problems in mechanics, using energy ideas makes it easier to solve a variety of problems in electricity.

When a charged particle moves in an electric field, the field exerts a force that can do *work* on the particle. This work can be expressed in terms of electric potential energy.

Just as gravitational potential energy depends on the height of a mass above the earth's surface, electric potential energy depends on the position of the charged particle in the electric field. We'll use a new concept called *electric potential*, or simply *potential*, to describe electric potential energy. In circuits, a difference in potential from one point to another is often called *voltage*. The concepts of potential and voltage are crucial to understanding how electric circuits work and have equally important applications to electron beams used in cancer radiotherapy, high-energy particle accelerators, and many other devices.

Electric potential energy

The concepts of work, potential energy, and conservation of energy proved to be extremely useful in our study of mechanics. In this section we'll show that these concepts are just as useful for understanding and analyzing electrical interactions. Let's begin by reviewing three essential points from mechanics. First, when a force $\boldsymbol{F}$ acts on a particle that moves from point a to point b, the work $W_{a\to b}$ done by the force is given by a *line integral:*

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F\cos\phi \, dl \quad \text{(work done by a force)}$$

where $d\boldsymbol{l}$ is an infinitesimal displacement along the particle's path and ϕ is the angle between vectors $\boldsymbol{F}$ and $d\boldsymbol{l}$ at each point along the path.

Second, if the force $\boldsymbol{F}$ is *conservative*, the work done by $\boldsymbol{F}$ can always be expressed in terms of a **potential energy** U. When the particle moves from a point where the potential energy is U_a to a point where it is U_b, the change in potential energy is $\Delta U = U_b - U_a$ and

Work done by a conservative force

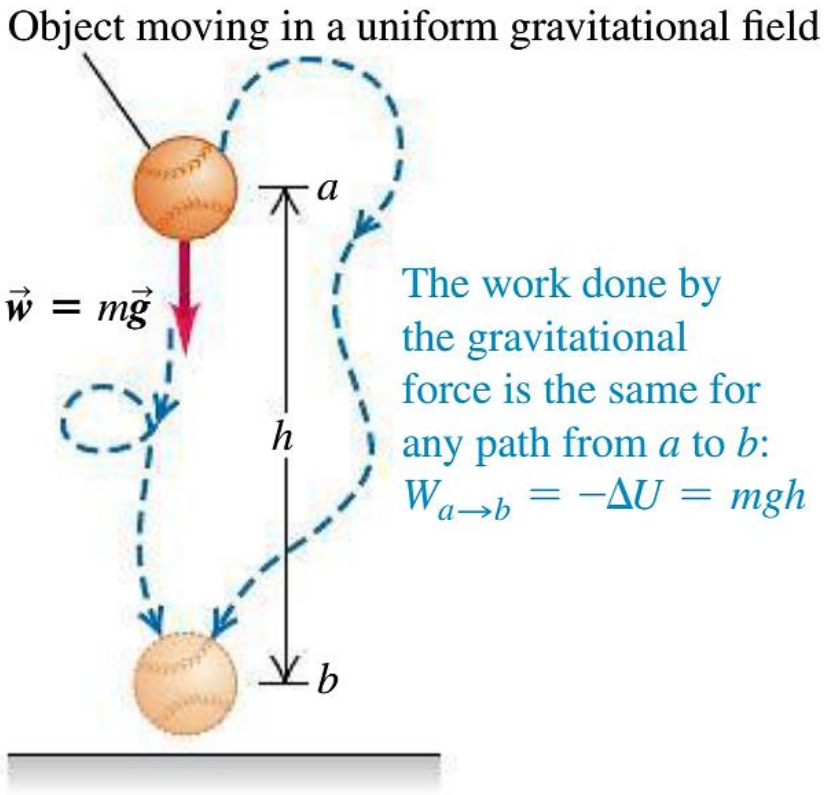
$$W_{a \to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

Potential energy at initial position

Potential energy at final position

Negative of change in potential energy

When $W_{a \to b}$ is positive, U_a is greater than U_b, ΔU is negative, and the potential energy *decreases*. That's what happens when a baseball falls from a high point (a) to a lower point (b) under the influence of the earth's gravity; the force of gravity does positive work, and the gravitational potential energy decreases (see the Figure). When a tossed ball is moving upward, the gravitational force does negative work during the ascent, and the potential energy increases.



The work done on a baseball moving in a uniform gravitational field.

Third, the work–energy theorem says that the change in kinetic energy $\Delta K = K_b - K_a$ during a displacement equals the *total* work done on the particle. If only conservative forces do work, then the Equation gives the total work, and $K_b - K_a = -(U_b - U_a)$. We usually write this as

$$K_a + U_a = K_b + U_b$$

That is, the total mechanical energy (kinetic plus potential) is *conserved* under these circumstances.

It can be shown that the work of the electric field force when the charge moves in it **does not depend on the path**, **but depends only on the initial (*a*) and final position (*b*)** of the charge. Thus the electric force is really a *conservative* force. Electric potential energy of two point charges can be written as

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Electric potential energy of two point charges

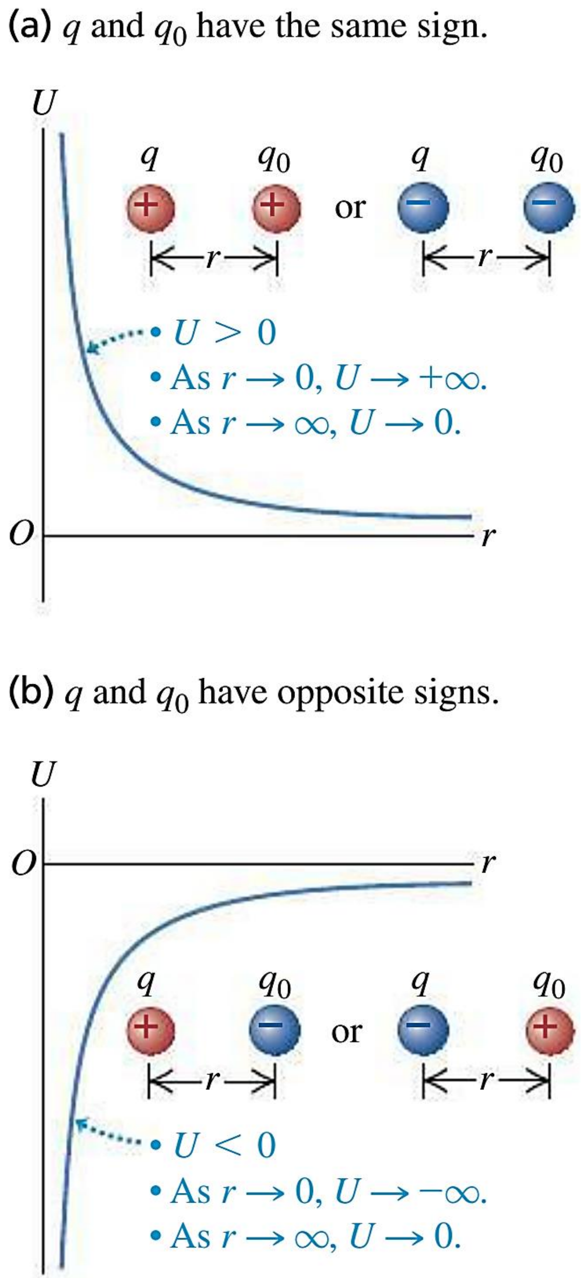
Values of two charges

Distance between two charges

Electric constant

The Equation is valid no matter what the signs of the charges q and q_0. The potential energy is positive if the charges q and q_0 have the same sign (see the Figure a) and negative if they have opposite signs (Fig. b).

Potential energy is always defined relative to some reference point where $U = 0$. In the Equation, U is zero when q and q_0 are infinitely far apart and $r = \infty$. Therefore U represents the work that would be done on the test charge q_0 by the field of q if q_0 moved from an initial distance r to infinity. If q and q_0 have the same sign, the interaction is repulsive, this work is positive, and U is positive at any finite separation (Fig. a). If the charges have opposite signs, the interaction is attractive, the work done is negative, and U is negative (Fig. b).



Graphs of the potential energy U of two point charges q and q_0 versus their separation r.

We emphasize that the potential energy U given by the Equation is a *shared* property of the two charges. If the distance between q and q_0 is changed from r_a to r_b, the change in potential energy is the same whether q is held fixed and q_0 is moved or q_0 is held fixed and q is moved. For this reason, we never use the phrase "the electric potential energy *of* a point charge." (Likewise, if a mass m is at a height h above the earth's surface, the gravitational potential energy is a shared property of the mass m and the earth.)

Interpreting electric potential energy

As a final comment, here are two viewpoints on electric potential energy. We have defined it in terms of the work done *by the electric field* on a charged particle moving in the field, just as in earlier we defined potential energy in terms of the work done by gravity or by a spring. When a particle moves from point a to point b, the work done on it by the electric field is $W_{a \to b} = U_a - U_b$. Thus the potential-energy difference $U_a - U_b$ equals *the work that is done by the electric force when the particle moves from a to b*. When U_a is greater than U_b, the field does positive work on the particle as it "falls" from a point of higher potential energy (a) to a point of lower potential energy (b).

An alternative but equivalent viewpoint is to consider how much work we would have to do to "raise" a particle from a point b where the potential energy is U_b to a point a where it has a greater value U_a (pushing two positive charges closer together, for example). To move the particle slowly (so as not to give it any kinetic energy), we need to exert an additional external force $\boldsymbol{F}_{\text{ext}}$ that is equal and opposite to the electric-field force and does positive work. The potentialenergy difference $U_a - U_b$ is then defined as *the work that must be done by an external force to move the particle slowly from b to a against the electric force*.

Because $\boldsymbol{F}_{\text{ext}}$ is the negative of the electric-field force and the displacement is in the opposite direction, this definition of the potential difference $U_a - U_b$ is equivalent to that given above. This alternative viewpoint also works if U_a is less than U_b, corresponding to "lowering" the particle; an example is moving two positive charges away from each other. In this case, $U_a - U_b$ is again equal to the work done by the external force, but now this work is negative.

We will use both of these viewpoints in the next section to interpret what is meant by electric *potential*, or potential energy per unit charge.

Electric potential

In previous section we looked at the potential energy U associated with a test charge q_0 in an electric field. Now we want to describe this potential energy on a "per unit charge" basis, just as electric field describes the force per unit charge on a charged particle in the field. This leads us to the concept of *electric potential,* often called simply *potential*. This concept is very useful in calculations involving energies of charged particles. It also facilitates many electric-field calculations because electric potential is closely related to the electric field $\boldsymbol{E}$. When we need to determine an electric field, it is often easier to determine the potential first and then find the field from it.

Potential is *potential energy per unit charge*. We define the potential V at any point in an electric field as the potential energy U *per unit charge* associated with a test charge q_0 at that point:

$$V = \frac{U}{q_0} \quad \text{or} \quad U = q_0 V$$

Potential energy and charge are both scalars, so potential is a scalar. From this Equation its units are the units of energy divided by those of charge. The SI unit of potential, called one **volt** (1 V) in honor of the Italian electrical experimenter Alessandro Volta (1745–1827), equals 1 joule per coulomb:

$$1\ \text{V} = 1\ \text{volt} = 1\ \text{J/C} = 1\ \text{joule/coulomb}$$

Let's put the Equation $W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$, which equates the work done by the electric force during a displacement from *a* to *b* to the quantity $-\Delta U = -(U_b - U_a)$, on a "work per unit charge" basis. We divide this equation by q_0, obtaining

$$\frac{W_{a\to b}}{q_0} = -\frac{\Delta U}{q_0} = -\left(\frac{U_b}{q_0} - \frac{U_a}{q_0}\right) = -(V_b - V_a) = V_a - V_b$$

where $V_a = U_a/q_0$ is the potential energy per unit charge at point *a* and similarly for V_b. We call V_a and V_b the *potential at point a* and *potential at point b*, respectively. Thus the work done per unit charge by the electric force when a charged body moves from *a* to *b* is equal to the potential at *a* minus the potential at *b*.

The difference $V_a - V_b$ is called the *potential of a with respect to b*; we sometimes abbreviate this difference as $V_{ab} = V_a - V_b$ (note the order of the subscripts). This is often called the potential difference between *a* and *b*, but that's ambiguous unless we specify which is the reference point. In electric circuits, which we will analyze in later chapters, the potential difference between two points is often called **voltage** (see the Figure below). The last Equation then states: **V_{ab}, the potential (in V) of *a* with respect to *b*, equals the work (in J) done by the electric force when a UNIT (1-C) charge moves from *a* to *b*.**

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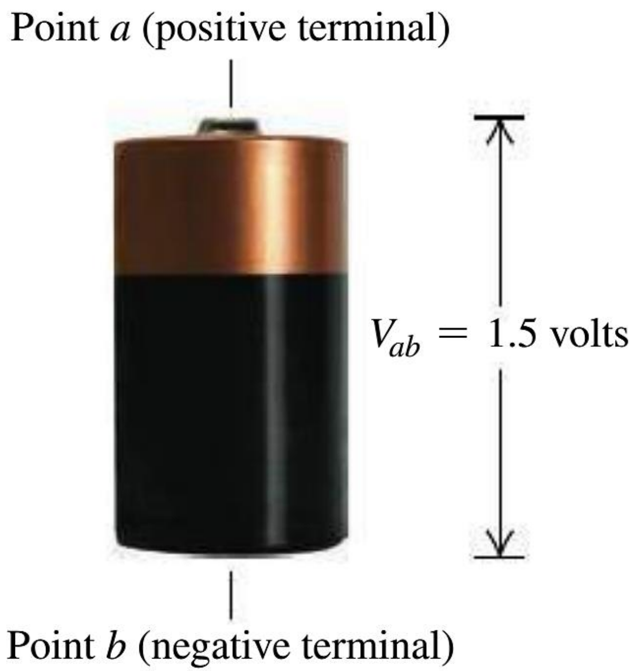


The voltage of this battery equals the difference in potential $V_{ab} = V_a - V_b$ between its positive terminal (point *a*) and its negative terminal (point *b*).

Calculating electric potential due to a point charge

To find the potential *V* due to a single point charge *q*, we divide the Eq. for potential energy *U* by q_0:

Electric potential due to a point charge

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Value of point charge

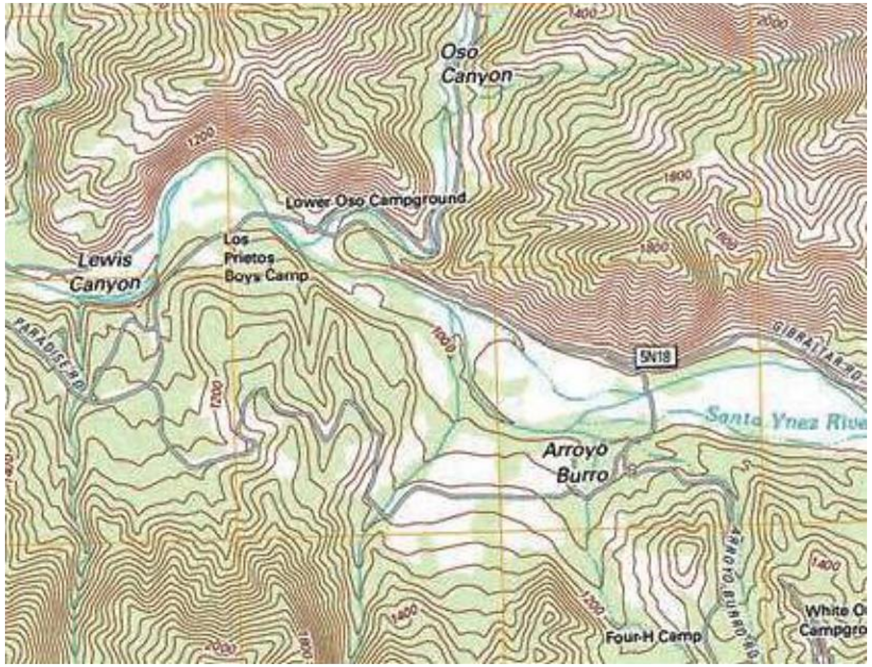
Distance from point charge to where potential is measured

Electric constant

If q is positive, the potential that it produces is positive at all points; if q is negative, it produces a potential that is negative everywhere. In either case, V is equal to zero at $r = \infty$, an infinite distance from the point charge. Note that potential, like electric field, is independent of the test charge q_0 that we use to define it.

Equipotential surfaces

Field lines help us visualize electric fields. In a similar way, the potential at various points in an electric field can be represented graphically by *equipotential surfaces*. These use the same fundamental idea as topographic maps like those used by hikers and mountain climbers (see the Figure). On a topographic map, contour lines are drawn through points that are all at the same elevation. Any number of these could be drawn, but typically only a few contour lines are shown at equal spacings of elevation. If a mass m is moved over the terrain along such a contour line, the gravitational potential energy mgy does not change because the elevation y is constant. Thus contour lines on a topographic map are really curves of constant gravitational potential energy. Contour lines are close together where the terrain is steep and there are large changes in elevation over a small horizontal distance; the contour lines are farther apart where the terrain is gently sloping. A ball allowed to roll downhill will experience the greatest downhill gravitational force where contour lines are closest together.



Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.

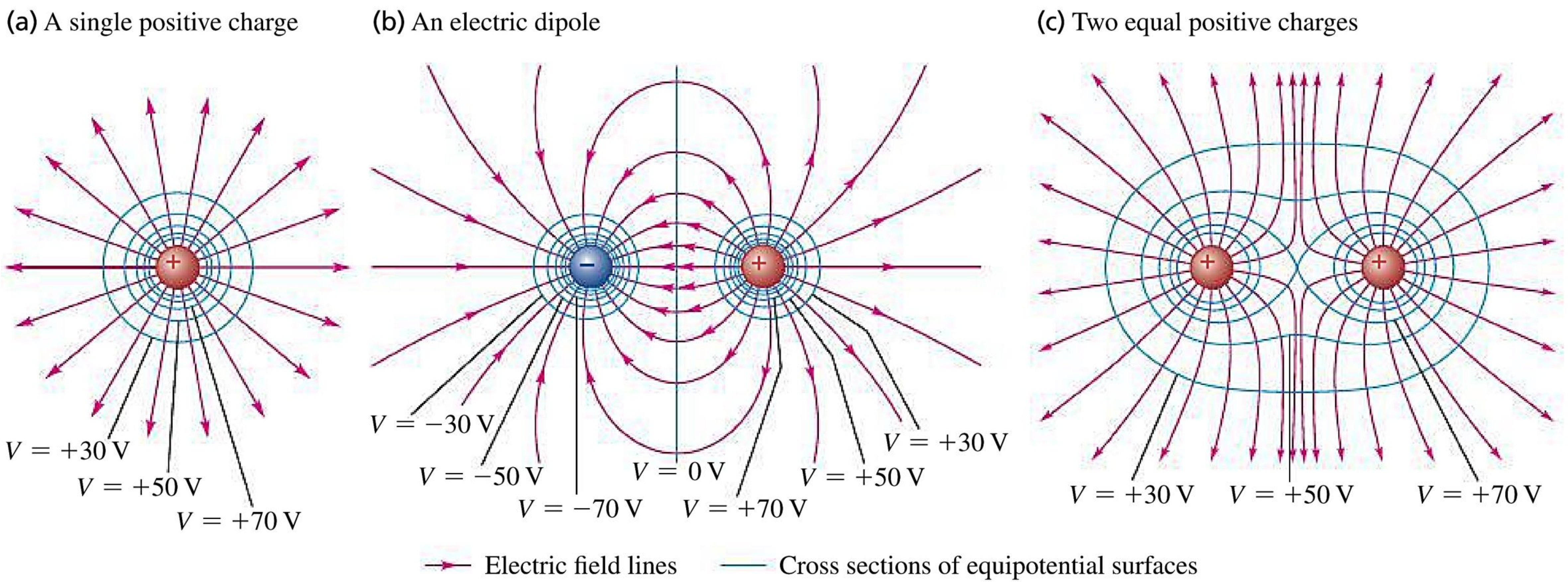
By analogy to contour lines on a topographic map, an **equipotential surface** is a three-dimensional surface on which the *electric potential* V is the same at every point. If a test charge q_0 is moved from point to point on such a surface, the *electric* potential energy q_0V remains constant. In a region where an electric field is present, we can construct an equipotential surface through any point. In diagrams we usually show only a few representative equipotentials, often with equal potential differences between adjacent surfaces. No point can be at two different potentials, so equipotential surfaces for different potentials can never touch or intersect.

Equipotential surfaces and field lines

Because potential energy does not change as a test charge moves over an equipotential surface, the electric field can do no work on such a charge. It follows that $\boldsymbol{E}$ must be perpendicular to the surface at every point so that the electric force $q_0\boldsymbol{E}$ is always perpendicular to the displacement of a charge moving on the surface. **Field lines and equipotential surfaces are always mutually perpendicular.** In general, field lines are curves, and equipotentials are curved surfaces. For the special case of a *uniform* field, in which the field lines are straight, parallel, and equally spaced, the equipotentials are parallel *planes* perpendicular to the field lines.

The Figure shows three arrangements of charges. The field lines in the plane of the charges are represented by red lines, and the intersections of the equipotential surfaces with this plane (that is, cross sections of these surfaces) are shown as blue lines. The actual equipotential surfaces are three-dimensional. At each crossing of an equipotential and a field line, the two are perpendicular.

In the Figure we have drawn equipotentials so that there are equal potential differences between adjacent surfaces.



Cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for assemblies of point charges. There are equal potential differences between adjacent surfaces.

In regions where the magnitude of $\boldsymbol{E}$ is large, the equipotential surfaces are close together because the field does a relatively large amount of work on a test charge in a relatively small displacement. This is the case near the point charge in Fig. a or between the two point charges in Fig. b; note that in these regions the field lines are also closer together. This is directly analogous to the downhill force of gravity being greatest in regions on a topographic map where contour lines are close together.

Conversely, in regions where the field is weaker, the equipotential surfaces are farther apart; this happens at larger radii in Fig. a, to the left of the negative charge or the right of the positive charge in Fig. b, and at greater distances from both charges in Fig. c. (It may appear that two equipotential surfaces intersect at the center of Fig. c, in violation of the rule that this can never happen. In fact this is a single figure-8–shaped equipotential surface.)

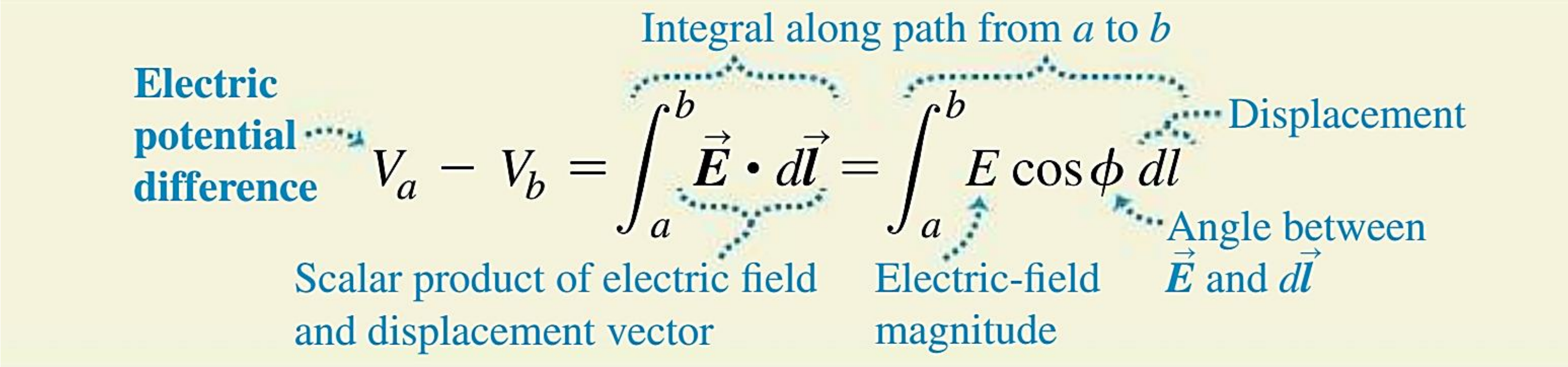
Pay attention: E need not be constant over an equipotential surface! On a given equipotential surface, the potential V has the same value at every point. In general, however, the electricfield magnitude E is *not* the same at all points on an equipotential surface. For instance, on equipotential surface "V = –30 V" in Fig. b, E is less to the left of the negative charge than it is between the two charges. On the figure-8–shaped equipotential surface in Fig. c, $E = 0$ at the middle point halfway between the two charges; at any other point on this surface, E is nonzero.

Finding electric potential from electric field

The force $\boldsymbol{F}$ on a test charge q_0 can be written as $\boldsymbol{F} = q_0\boldsymbol{E}$, so the work done by the electric force as the test charge moves from a to b is given by

$$W_{a\to b} = \int_a^b \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{l}} = \int_a^b q_0\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

If we divide this by q_0, we find



The Equation can be rewritten as

$$V_a - V_b = -\int_b^a \vec{E} \cdot d\vec{l}$$

This has a negative sign compared to the integral in previous Equation, and the limits are reversed; hence both these Equations are equivalent.

If we know ***E*** as a function of position, we can calculate *V* from these Equations.

These Equations show that the unit of potential difference (1 V) is equal to the unit of electric field (1 N/C) multiplied by the unit of distance (1 m). Hence the unit of electric field can be expressed as 1 *volt per meter* (1 V/m), as well as 1 N/C:

$$1 \text{ V/m} = 1 \text{ volt/meter} = 1 \text{ N/C} = 1 \text{ newton/coulomb}$$

In practice, the volt per meter is the usual unit of electric-field magnitude.

Potential gradient

Electric field and potential are closely related. Let's look at the Equation, restated here:

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$$

If we know $\vec{E}$ at various points, we can use this equation to calculate potential differences. In this section we show how to turn this around; if we know the potential V at various points, we can use it to determine $\vec{E}$. Regarding V as a function of the coordinates (x, y, z) of a point in space, we will show that the components of $\vec{E}$ are related to the *partial derivatives* of V with respect to x, y, and z.

In the Equation, $V_a - V_b$ is the potential of a with respect to b – that is, the change of potential encountered on a trip from b to a. We can write this as

$$V_a - V_b = \int_b^a dV = -\int_a^b dV$$

where dV is the infinitesimal change of potential accompanying an infinitesimal element $d\boldsymbol{l}$ of the path from b to a. So, we have

$$-\int_a^b dV = \int_a^b \vec{E} \cdot d\vec{l}$$

These two integrals must be equal for *any* pair of limits a and b, and for this to be true the *integrands* must be equal. Thus, for *any* infinitesimal displacement $d\vec{l}$,

$$-dV = \vec{E} \cdot d\vec{l}$$

To interpret this expression, we write $\vec{E}$ and $d\vec{l}$ in terms of their components: $\vec{E} = \hat{\imath}\, E_x + \hat{\jmath}\, E_y + \hat{k}\, E_z$ and $d\vec{l} = \hat{\imath}\, dx + \hat{\jmath}\, dy + \hat{k}\, dz$. Then

$$-dV = E_x dx + E_y dy + E_z dz$$

Suppose the displacement is parallel to the x-axis, so $dy = dz = 0$. Then $-dV = E_x dx$ or $E_x = -(dV/dx)_{y,\, z\ \text{constant}}$, where the subscript reminds us that only x varies in the derivative; recall that V is in general a function of x, y, and z. But this is just what is meant by the partial derivative $\partial V/\partial x$. The y- and z-components of $\vec{E}$ are related to the corresponding derivatives of V in the same way, so

Electric field components found from potential:

Each electric field component ...

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

... equals the negative of the corresponding partial derivative of electric potential function V.

This is consistent with the units of electric field being V/m. In terms of unit vectors we can write

Electric field vector found from potential:

$$\vec{\boldsymbol{E}} = -\left(\hat{\imath}\frac{\partial V}{\partial x} + \hat{\jmath}\frac{\partial V}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial V}{\partial z}\right)$$

Electric field

Partial derivatives of electric potential function V

The following operation is called the **gradient** of the function f:

$$\vec{\nabla} f = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{\boldsymbol{k}}\frac{\partial}{\partial z}\right) f$$

The operator denoted by $\vec{\nabla}$ is called "grad" or "del." Thus in vector notation,

$$\vec{\boldsymbol{E}} = -\vec{\nabla} V$$

This is read "$\vec{\boldsymbol{E}}$ is the negative of the gradient of V" or "$\vec{\boldsymbol{E}}$ equals negative grad V." The quantity $\vec{\nabla} V$ is called the *potential gradient*.

At each point, the potential gradient $\vec{\nabla} V$ points in the direction in which V *increases* most rapidly with a change in position. Hence at each point the direction of $\vec{E} = -\vec{\nabla} V$ is the direction in which V *decreases* most rapidly and is always perpendicular to the equipotential surface through the point.

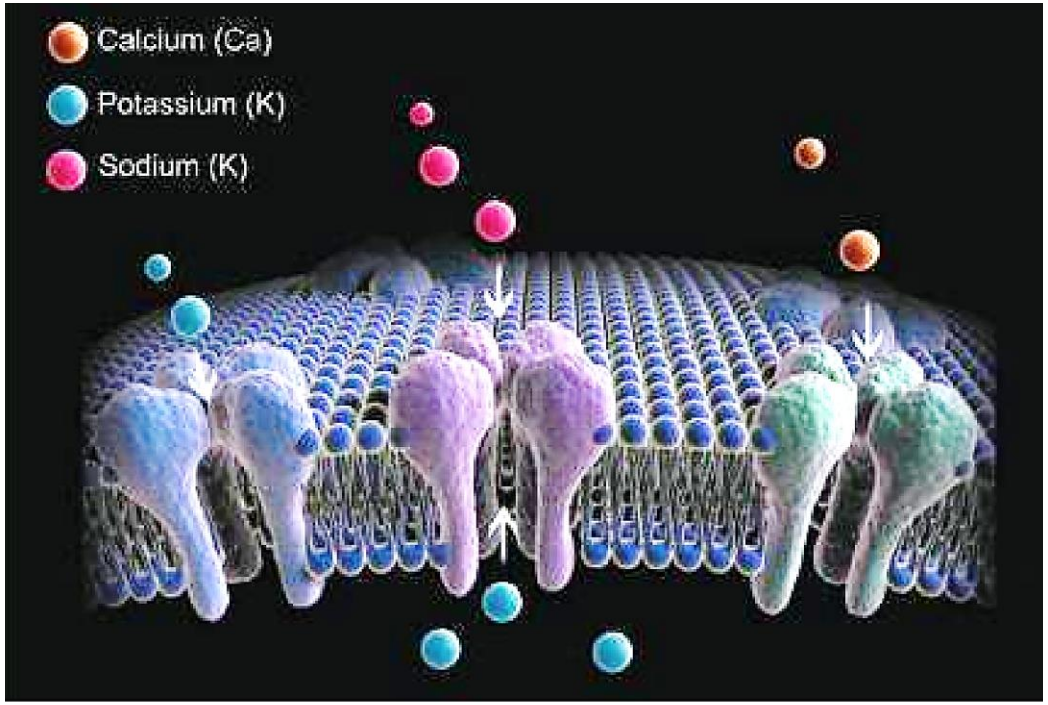
This agrees with our early observation that moving in the direction of the electric field means moving in the direction of decreasing potential. The Equation above doesn't depend on the particular choice of the zero point for V. If we were to change the zero point, the effect would be to change V at every point by the same amount; the derivatives of V would be the same. If $\boldsymbol{E}$ has a radial component E_r with respect to a point or an axis and r is the distance from the point or axis, the relationship corresponding to located above Equations is

$$E_r = -\frac{\partial V}{\partial r} \qquad \text{(radial electric field)}$$

Often we can compute the electric field caused by a charge distribution in either of two ways: directly, by adding the $\boldsymbol{E}$ fields of point charges, or by first calculating the potential and then taking its gradient to find the field. The second method is often easier because potential is a *scalar* quantity, requiring at worst the integration of a scalar function. Electric field is a *vector* quantity, requiring computation of components for each element of charge and a separate integration for each component. Thus, quite apart from its fundamental significance, potential offers a very useful computational technique in field calculations. If we know V as a function of position, we can calculate $\boldsymbol{E}$ from these Eqs. Deriving V from $\boldsymbol{E}$ requires integration, and deriving $\boldsymbol{E}$ from V requires differentiation.

Example: Potential gradient across a cell membrane

The interior of a human cell is at a lower electric potential V than the exterior. (The potential difference when the cell is inactive is about – 70 mV in neurons and about – 95 mV in skeletal muscle cells.) Hence there is a potential gradient ∇V that points from the *interior* to the *exterior* of the cell membrane, and an electric field $\boldsymbol{E} = -\nabla V$ that points from the *exterior* to the *interior*. This field affects how ions flow into or out of the cell through special channels in the membrane.



Electric field in the medium

So far, we have considered the electric field in a *vacuum*. Any medium reduces the effect of the electric field on the charge placed in it by a factor of ε (this reduces the value of the electric field $\boldsymbol{E}$ and the potential V.) The value ε is called the **permittivity** and has its own value for various substances. For example, the formulas for the field and potential at a distance r from the point charge q that creates the field have the form:

$$E = \frac{1}{4\pi\varepsilon\varepsilon_0} \cdot \frac{q}{r^2}, \qquad V = \frac{1}{4\pi\varepsilon\varepsilon_0} \cdot \frac{q}{r}.$$

Electron Volts

The magnitude e of the electron charge can be used to define a unit of energy that is useful in many calculations with atomic and nuclear systems. When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - U_b = q(V_a - V_b) = qV_{ab}$$

If charge q equals the magnitude e of the electron charge, 1.602×10^{-19} C, and the potential difference is $V_{ab} = 1$ V, the change in energy is

$$U_a - U_b = (1.602 \times 10^{-19}\ \text{C})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$$

This quantity of energy is defined to be 1 **electron volt** (1 eV):

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

The multiples meV, keV, MeV, GeV, and TeV are often used.

CAUTION **Electron volts vs. volts** Remember that the electron volt is a unit of energy, *not* a unit of potential or potential difference!

When a particle with charge e moves through a potential difference of 1 volt, the change in potential *energy* is 1 eV. If the charge is some multiple of e—say, Ne—the change in potential energy in electron volts is N times the potential difference in volts. For example, when an alpha particle, which has charge $2e$, moves between two points with a potential difference of 1000 V, the change in potential energy is $2(1000\ \text{eV}) = 2000\ \text{eV}$. To confirm this, we write

$$U_a - U_b = qV_{ab} = (2e)(1000\ \text{V}) = (2)(1.602 \times 10^{-19}\ \text{C})(1000\ \text{V})$$
$$= 3.204 \times 10^{-16}\ \text{J} = 2000\ \text{eV}$$

Although we defined the electron volt in terms of *potential* energy, we can use it for *any* form of energy, such as the kinetic energy of a moving particle. When we speak of a "one-million-electron-volt proton," we mean a proton with a kinetic energy of one million electron volts (1 MeV), equal to $(10^6)(1.602 \times 10^{-19}\ \text{J}) = 1.602 \times 10^{-13}\ \text{J}$. The Large Hadron Collider near Geneva, Switzerland, is designed to accelerate protons to a kinetic energy of 7 TeV (7×10^{12} eV).