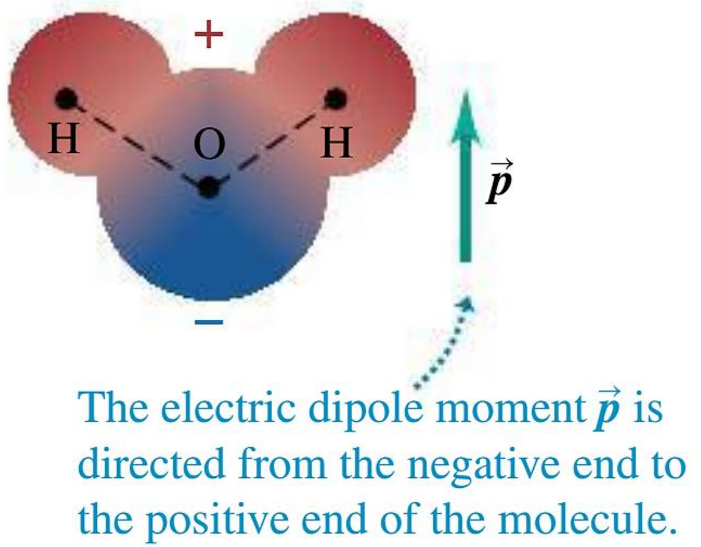
Electric dipoles

An **electric dipole** is a pair of point charges with equal magnitude and opposite sign (a positive charge q and a negative charge $-q$) separated by a distance d. Many physical systems, from molecules to TV antennas, can be described as electric dipoles.

Figure shows a molecule of water (H_2O), which in many ways behaves like an electric dipole. The water molecule as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge; the result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole. The effect is equivalent to shifting one electron only about 4×10^{-11} m (about the radius of a hydrogen atom), but the consequences of this shift are profound.

Water is an excellent solvent for ionic substances such as table salt (sodium chloride, NaCl) precisely because the water molecule is an electric dipole. When dissolved in water, salt dissociates into a positive sodium ion (Na^+) and a negative chlorine ion (Cl^-), which tend to be attracted to the negative and positive ends, respectively, of water molecules; this holds the ions in solution. If water molecules were not electric dipoles, water would be a poor solvent, and almost all of the chemistry that occurs in aqueous solutions would be impossible. This includes all of the biochemical reactions that occur in all of the life on earth. In a very real sense, your existence as a living being depends on electric dipoles!



A water molecule, showing positive charge as red and negative charge as blue. So a water molecule is an example of an electric dipole. The large electric dipole moment of water makes it an excellent solvent.

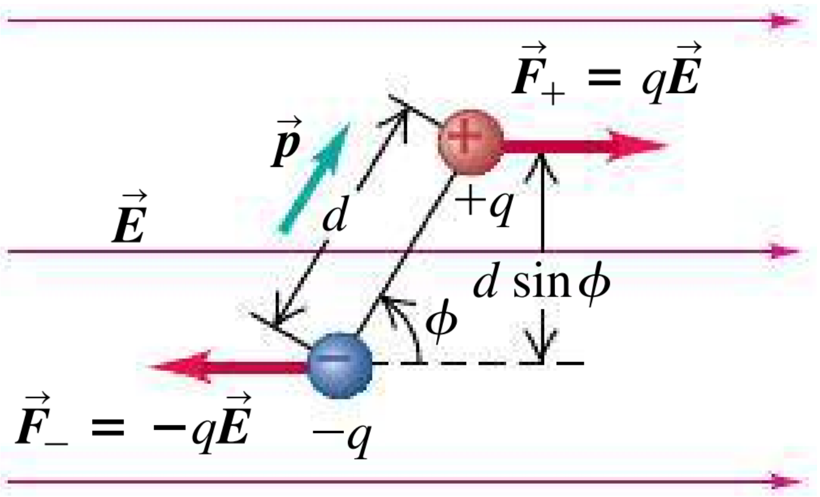
We examine two questions about electric dipoles. First, what forces and torques does an electric dipole experience when placed in an external electric field (that is, a field set up by charges outside the dipole)? Second, what electric field does an electric dipole itself produce? These two situations reveal, respectively, the *passive* and *active properties* of the dipole.

Force and torque on an electric dipole

To start with the first question, let's place an electric dipole in a *uniform* external electric field $\boldsymbol{E}$, as shown in Figure. Both forces $\boldsymbol{F}_+$ and $\boldsymbol{F}_-$ on the two charges have magnitude qE, but their directions are opposite, and they add to zero. *The net force on an electric dipole in a uniform external electric field is zero.*

However, the two forces don't act along the same line, so their *torques* don't add to zero. We calculate torques with respect to the center of the dipole. Let the angle between the electric field $\boldsymbol{E}$ and the dipole axis be ϕ; then the lever arm for both $\boldsymbol{F}_+$ ***and*** $\boldsymbol{F}_-$ is $(d/2)\sin\phi$. The torque of $\boldsymbol{F}_+$ and the torque of $\boldsymbol{F}_-$ both have the same magnitude of $(qE)(d/2)\sin\phi$, and both torques tend to rotate the dipole clockwise (that is, $\boldsymbol{\tau}$ is directed into the page in Fig. 21.31). Hence the magnitude of the net torque is twice the magnitude of either individual torque:

$\boldsymbol{\tau} = (qE)(d\sin\phi)$, where $d\sin\phi$ is the perpendicular distance between the lines of action of the two forces.



The net force on this electric dipole is zero, but there is a torque directed into the page that tends to rotate the dipole clockwise.

The product of the charge q and the separation d is the magnitude of a quantity called the **electric dipole moment,** denoted by p: $p = qd$ (magnitude of electric dipole moment)

The units of p are charge times distance (C · m). For example, the magnitude of the electric dipole moment of a water molecule is $p = 6.13 \times 10^{-30}$ C · m.

Caution: the symbol p has multiple meanings. Do not confuse dipole moment with momentum or pressure. There aren't as many letters in the alphabet as there are physical quantities, so some letters are used several times. The context usually makes it clear what we mean, but be careful.

We further define the electric dipole moment to be a *vector* quantity $\boldsymbol{p}$. The magnitude of $\boldsymbol{p}$ is given by the Equation above, and its direction is along the dipole axis from the negative charge to the positive charge as shown in the Figure. In terms of p, Equation for the magnitude τ of the torque exerted by the field becomes

Magnitude of electric field $\vec{E}$

Magnitude of torque on an electric dipole

$$\tau = pE\sin\phi$$

Angle between $\vec{p}$ and $\vec{E}$

Magnitude of electric dipole moment $\vec{p}$

Since the angle ϕ in the Figure is the angle between the directions of the vectors $\boldsymbol{p}$ and $\boldsymbol{E}$, this is reminiscent of the expression for the magnitude of the *vector product*. Hence we can write the torque on the dipole in vector form as

Electric dipole moment

Vector torque on an electric dipole

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{p}} \times \vec{\boldsymbol{E}}$$

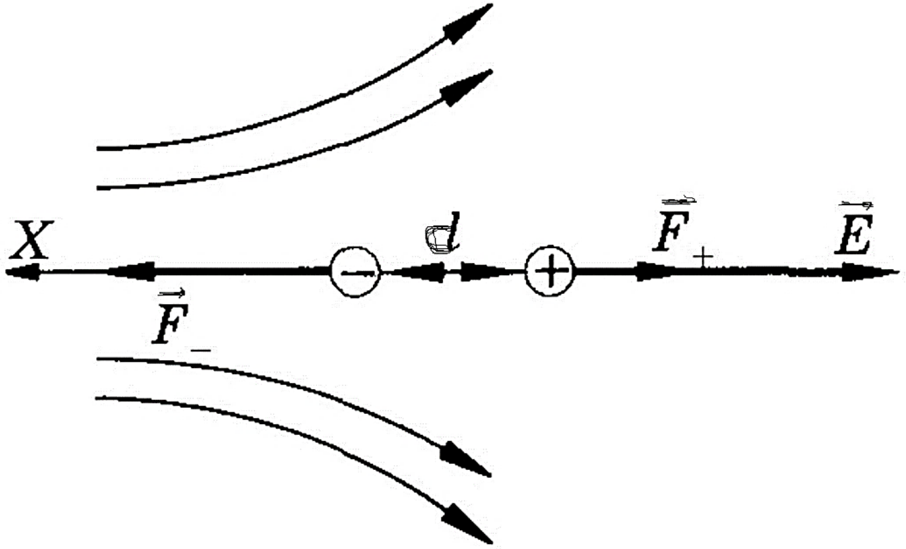
Electric field

You can use the right-hand rule for the vector product to verify that in the situation shown in the Figure, $\boldsymbol{\tau}$ is directed into the page. The torque is greatest when $\boldsymbol{p}$ and $\boldsymbol{E}$ are perpendicular and is zero when they are parallel or antiparallel. The torque always tends to turn $\boldsymbol{p}$ to line it up with $\boldsymbol{E}$. The position $\phi = 0$, with $\boldsymbol{p}$ parallel to $\boldsymbol{E}$, is a position of stable equilibrium, and the position $\phi = \pi$, with $\boldsymbol{p}$ and $\boldsymbol{E}$ antiparallel, is a position of unstable equilibrium.

Let's now consider a dipole in an inhomogeneous electric field. Assume that the dipole is located along an electric field line whose direction is opposite to the *OX*-axis (see figure). On the dipole act forces

$$\vec{E}_+ = q\vec{E}_+ \text{ и } \vec{E}_- = -q\vec{E}_- \,,$$

where $\boldsymbol{E}_+$ and $\boldsymbol{E}_-$ is the electric field, respectively, at the location of the positive and negative charges (since the density of the lines in Fig. on the left is greater than on the right, then $\boldsymbol{E}_- > \boldsymbol{E}_+$). The value of the net force:



Forces acting on a dipole in a non-uniform electric field.

$$F = F_- - F_+ = qE_- - qE_+ = q(E_- - E_+).$$

This force will move the dipole to the left - pull it into the area of a stronger field. We introduce a relation $(E_- - E_+)/d$ that characterizes the average change in the field value per unit length of the dipole. Since the separation d is usually small, it can be approximated as

$$(E_- - E_+)/d = dE/dx,$$

where *dE/dx* is the derivative of the field value in the direction of the *OX* axis, which is a measure of the non-uniformity of the electric field along the corresponding direction. It follows from this equation that

$$E_{-} - E_{+} = d\frac{dE}{dx},$$

then the formula for force can be represented as

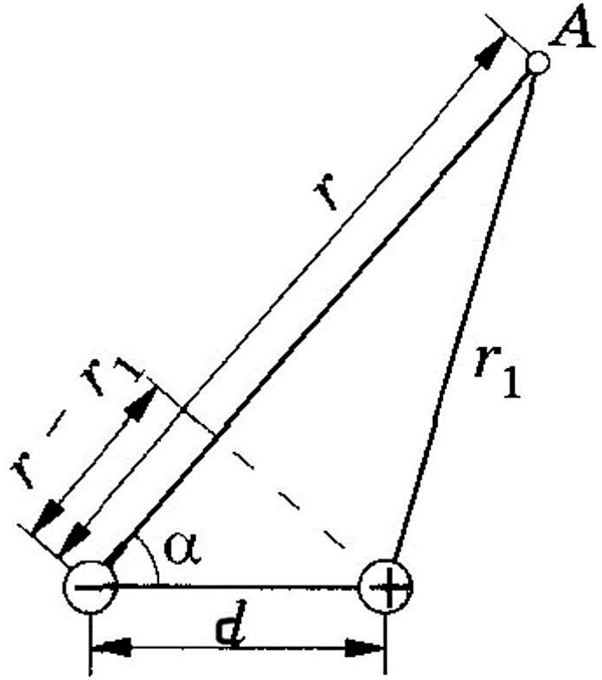
$$F = qd\frac{dE}{dx} = p \cdot \frac{dE}{dx}.$$

So, the dipole is affected by a force that depends on its electric dipole moment and the degree of non-uniformity of the field *dE/dx*.

Active properties of the dipole

So far, we have considered a dipole placed in an electric field, but the dipole itself is the source of the field. Let's now consider **some of the active properties** of the dipole.

Let's write an expression for the electric potential of the field created by the dipole at some point *A*, distant from the charges at distances r and r_1, respectively (see the figure):



$$V = \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r}\right) = \frac{q}{4\pi\varepsilon\varepsilon_0} \cdot \frac{r - r_1}{rr_1}.$$

Suppose that $d << r$, $d << r_1$ then $r \approx r_1$ and

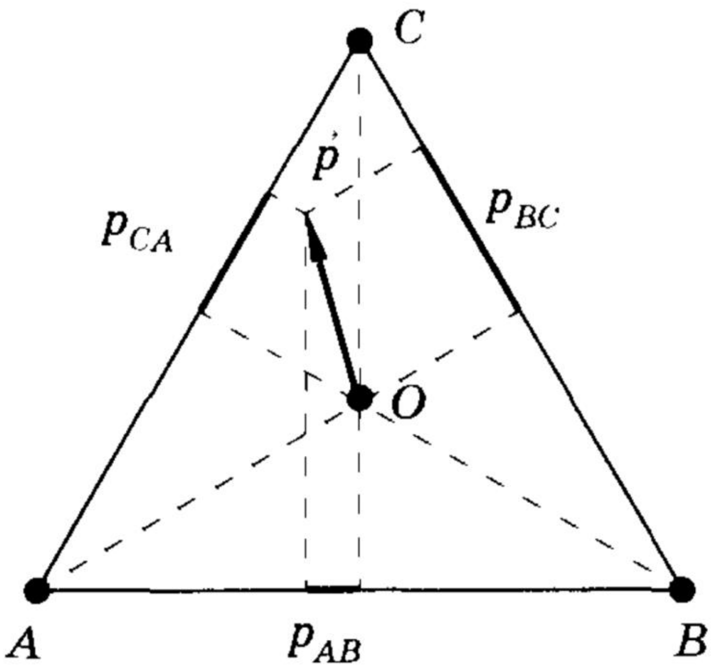
$$rr_1 = r^2 \,,\; r - r_1 = d\cos\alpha,$$

where α is the angle between the vector $\boldsymbol{p}$ and the direction from the dipole to point A (see the Fig.). Thus, we get the expression for the potential:

$$V = \frac{qd\cos\alpha}{4\pi\varepsilon\varepsilon_0 r^2} = \frac{1}{4\pi\varepsilon\varepsilon_0} \cdot \frac{p\cos\alpha}{r^2}.$$

Note here the result that the potential of a system of two charges (i.e., a dipole) is inversely proportional to r in the second degree, $V \sim 1/r^2$.

Let the dipole that creates the electric field be located in the center of an equilateral triangle ABC (see the Figure). Using the above expression for the potential, we can prove (we will not draw a complete conclusion here) that the electric voltages on the sides of this triangle are related as projections $\boldsymbol{p}$ on its sides:



$$U_{AB} : U_{BC} : U_{CA} = p_{AB} : p_{BC} : p_{CA}.$$

where each voltage is a difference of potentials:

$$U_{AB} = V_A - V_B,$$

and so on.

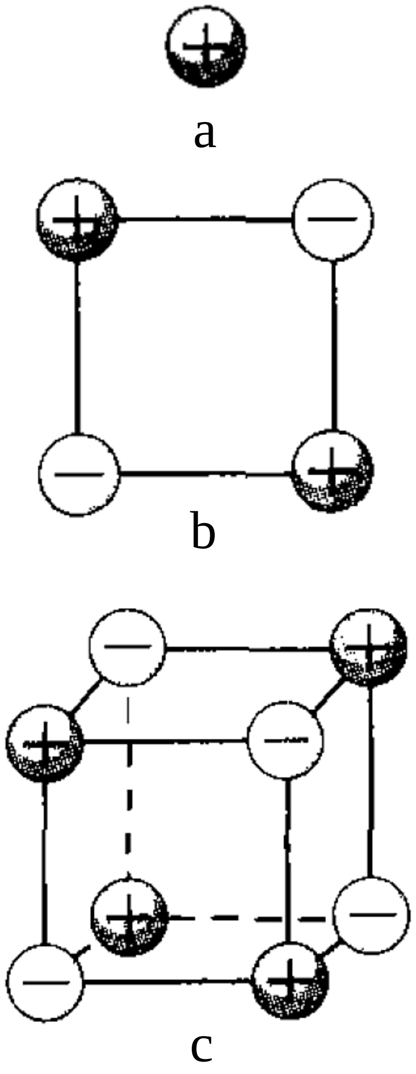
The concept of multipoles

A dipole is a special case of a system of electric charges that have a certain symmetry. You can specify more examples of symmetric charge systems (see the Figure). The general name of such charge distributions is *electric multipoles*. They are of different orders (l= 0, 1, 2, etc.), the number of multipole charges is determined by the expression $N=2^l$. Thus, the multipole of order zero ($2^0 = 1$) is a single point charge – *monopole* (Fig. a), a multipole of first order ($2^1 = 2$) is a *dipole*, a multipole of second order ($2^2 = 4$) – *quadrupole* (Fig. b), multipole of third order ($2^3 = 8$) – *octupole* (Fig. c), etc.

The potential of the multipole field decreases at significant distances from it ($r >> d$, where d is the size of the multipole) proportionally to $1/r^{l+1}$. So, for the one charge (l = 0) $V \sim 1/r$ (we proved this in the previous lecture), for the dipole (l = 1) $V \sim 1/r^2$ (as we have just proved), for a quadrupole (l = 2) $V \sim 1/r^3$, etc.

If the charge is distributed in a certain area of space, then the potential of the electric field outside the charge system can be represented as an approximate series:

$$V = \frac{f_1}{r} + \frac{f_2}{r^2} + \frac{f_3}{r^3} + \ldots.$$



Samples of multipoles: (a) monopole, (b) quadrupole, (c) octupole.

Here r is the distance from the charge system to point A with potential V; f_1, f_2, f_3 are some functions that depend on the type of multipole, the values of its charges, and the direction to point A. The first term in this formula corresponds to a monopole, the second to a dipole, the third to a quadrupole, and so on.

In the case of a neutral system of charges the first term is equal to zero.

If r is very large, then you can ignore all the terms of the series starting from the third. Then the potential of this entire arbitrary system of charges can be approximated by the potential of the dipole.

Physical basis of electrocardiography

Living tissues are a source of electrical potentials (biopotentials.) Registration of tissue and organ biopotentials for diagnostic (or research) purposes is called *electrography*. This general term is used relatively rarely, more common are the specific names of the corresponding diagnostic methods: *electrocardiography* (ECG or EKG) – registration of biopotentials that occur in the heart muscle when it is excited, *electromyography* (EMG) – a method for registering the bioelectric activity of muscles, *electroencephalography* – EEG) – a method for registering the bioelectric activity of the brain, etc.

In most cases, biopotentials are registered by electrodes not directly from the organ (heart, brain), but from other "neighboring" tissues in which electric fields are created by this organ. In clinical terms, this greatly simplifies the registration procedure itself, making it safe and simple. The physical approach to electrography is to create (or select) a model of an electric generator that corresponds to the pattern of registered potentials. In this regard, two fundamental theoretical problems arise here.

The direct problem is to calculate the potential in the measurement area based on the specified characteristics of the electric generator (model).

The inverse problem is to calculate the characteristics of an electric generator based on the measured potential.

Further specific consideration of physical issues of electrography will be done on the example of electrocardiography. One of the main tasks of theoretical electrocardiography is to calculate the distribution of the transmembrane potential of heart muscle cells by potentials measured outside the heart. However, even theoretically, such an **inverse problem** cannot be solved, since the same external manifestation of heart biopotentials can be at different internal distribution.

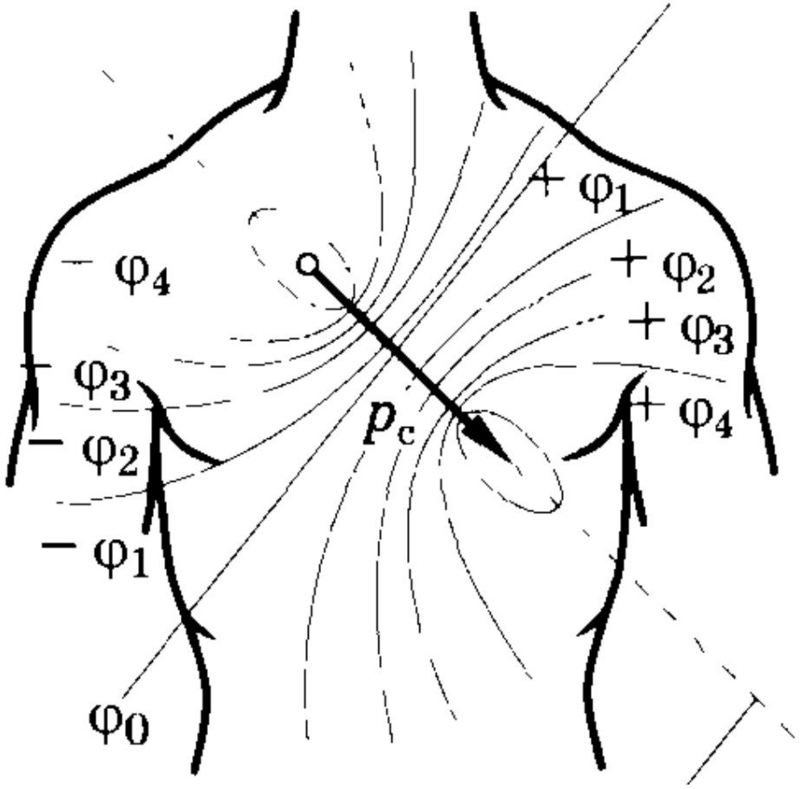
The physical (biophysical) approach to elucidating the relationship between heart biopotentials and their external manifestation is to model the sources of these biopotentials. The whole heart is represented in electrical terms as some equivalent electrical generator, either purely speculative (hypothetical), or as a real device as a collection of electrical sources in a conductor that has the shape of a human body.

Applying considerations similar to those made above for multipoles and a system of arbitrarily distributed charges for the heart, it can be shown that the electrical properties of the heart can be modeled, in the first approximation, by representing the heart **as a dipole**. In other words, in the multipole equivalent generator of the heart, the main part of the potential on the surface of the human body is introduced by its **dipole component**.

The dipole view of the heart is the basis of the theory of leads created by Dutch physician and physiologist Willem Einthoven (1860 –1927). According to it, **the heart is a dipole with a dipole moment that turns, changes its position and the point of application** (changing the point of application of this vector is often neglected) **during the cardiac cycle**.

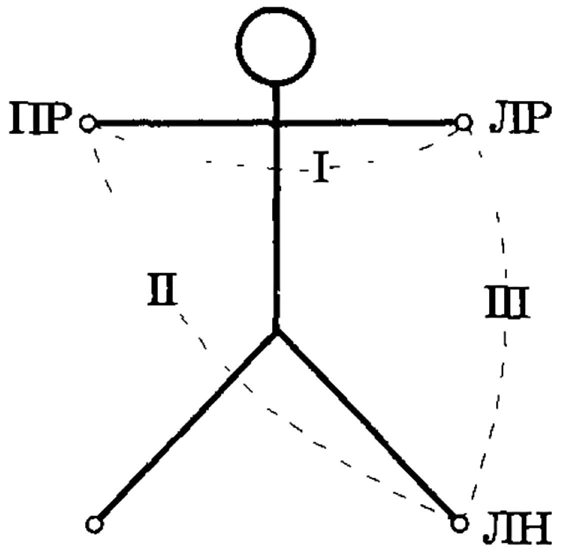
The Figure shows the positions of the vector ***p*** and equipotential lines for the moment of time when the dipole moment is maximal; this corresponds to the *R* wave on the electrocardiogram.

Einthoven proposed to measure the differences in heart biopotentials between the vertices of an equilateral triangle, which are approximately located in the right arm (RA, or ПР in Russian), left arm (LR or ЛР) and left leg (LL or ЛН) (see the next figure a). As we said earlier, if the triangle is equilateral and the heart is located in the center of the triangle, then by finding the ratio of potential differences between the vertices of the triangle, we will determine the relationship between the projections of the electric moment of the heart on the sides of the triangle, i.e. we will find the magnitude and direction of the vector ***p***. The Figure b schematically shows this triangle.

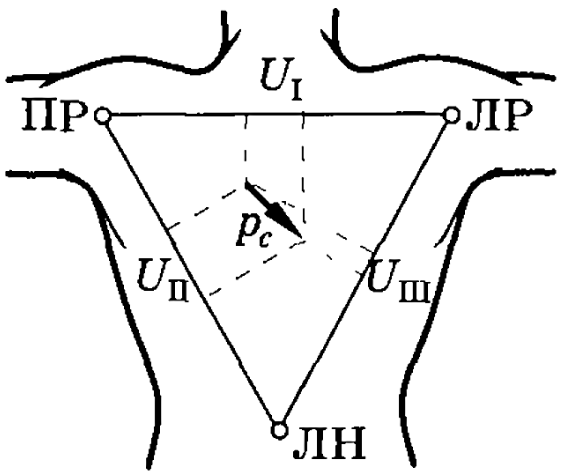


Electrical axis of the heart

Electric dipole moment $\boldsymbol{p}_c$ of a heart and lines of equal values of potential φ (*V*).



a



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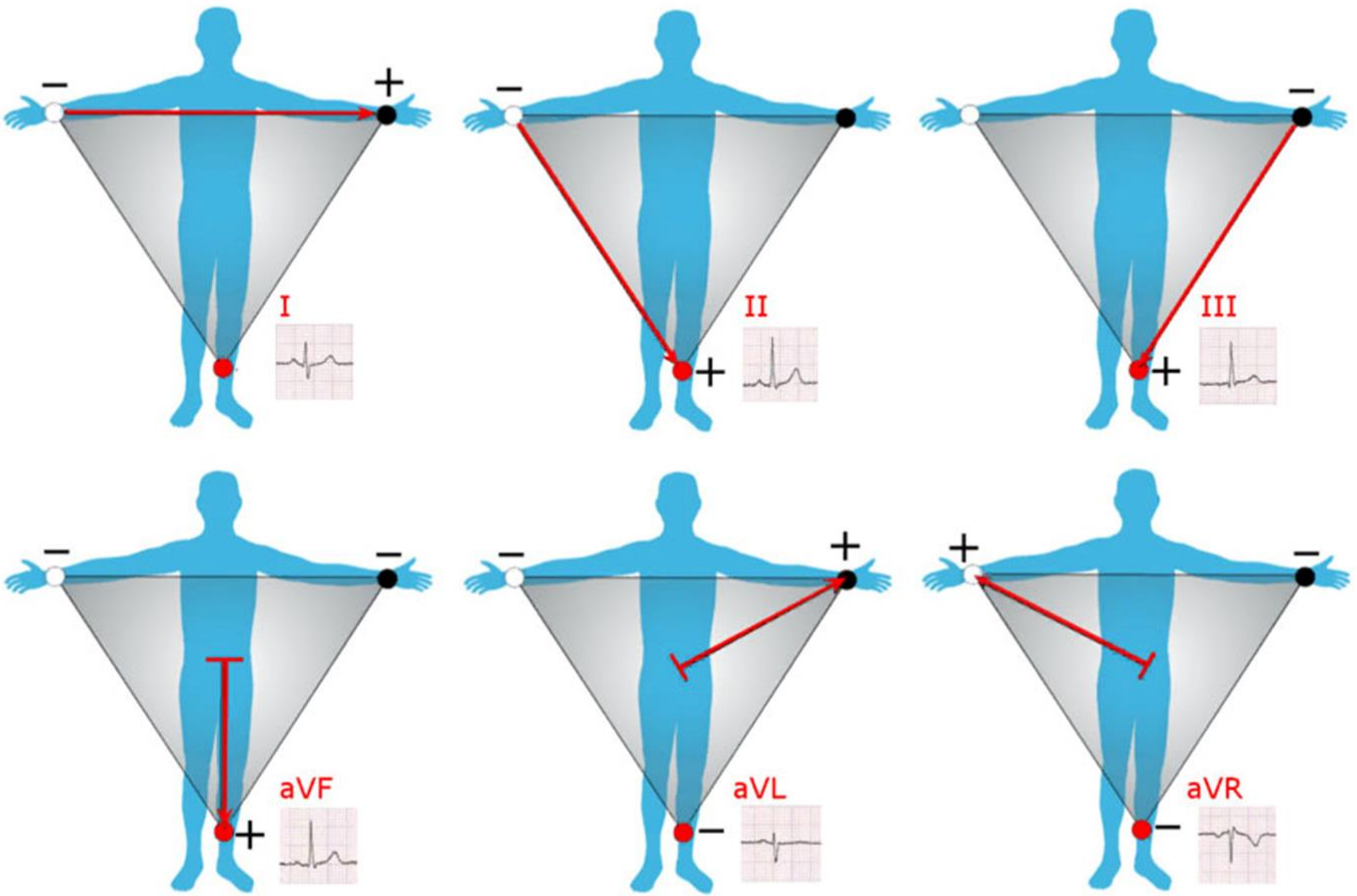
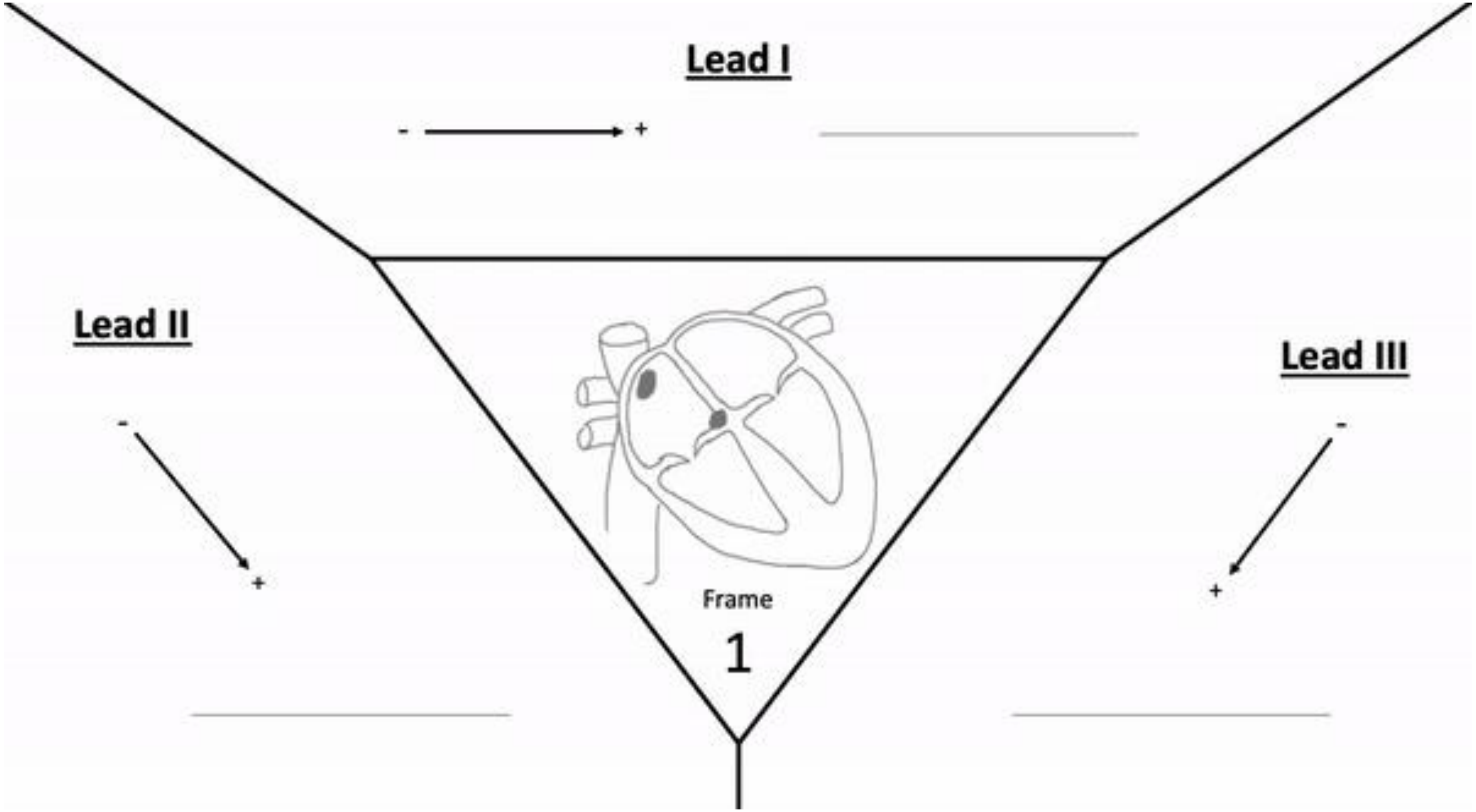


Graphical representation of Einthoven's triangle

In the terminology of physiologists, the difference of biopotentials registered between two points of the body is called a lead. Distinguish I lead (right shoulder – left shoulder), II lead (right shoulder – left leg) and III lead (left shoulder – left leg). Since the electric moment of the dipole – the heart – changes with time, the voltage dependences on time will be obtained in the leads, which are called electrocardiograms. The registered voltages are typically no greater than 1 mV = 10^{-3} V.

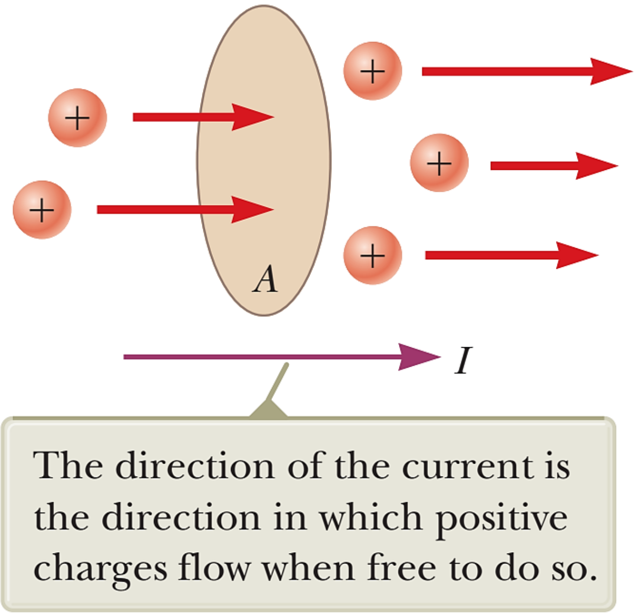


Formation of limb waveforms during a pulse

Electric current

In this section, we study the flow of electric charges through a piece of material. The amount of flow depends on both the material through which the charges are passing and the potential difference across the material. Whenever there is a net flow of charge through some region, an electric *current* is said to exist.

It is instructive to draw an analogy between water flow and current. The flow of water in a plumbing pipe can be quantified by specifying the amount of water that emerges from a faucet during a given time interval, often measured in liters per minute. A river current can be characterized by describing the rate at which the water flows past a particular location. For example, the flow over the brink at Niagara Falls is maintained at rates between 1400 m^3/s and 2800 m^3/s.



Charges in motion through an area *A*. The time rate at which charge flows through the area is defined as the current *I*.

To define current quantitatively, suppose charges are moving perpendicular to a surface of area *A* as shown in the Figure. (This area could be the cross-sectional area of a wire, for example.) The **current** is defined as the rate at which charge flows through this surface. If ΔQ is the amount of charge that passes through this surface in a time interval Δt, the **average current** I_{avg} is equal to the charge that passes through *A* per unit time:

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

If the rate at which charge flows varies in time, the current varies in time; we define the **instantaneous current** I as the limit of the average current as $\Delta t \rightarrow 0$:

$$I \equiv \frac{dQ}{dt}$$

The SI unit of current is the **ampere** (A):

$$1\text{ A} = 1\text{ C/s}.$$

That is, 1 A of current is equivalent to 1 C of charge passing through a surface in 1 s.

The charged particles passing through the surface in the Figure can be positive, negative, or both. It is conventional to assign to the current the same direction as the flow of positive charge. In electrical conductors such as copper or aluminum, the current results from the motion of negatively charged electrons. Therefore, in an ordinary conductor, the direction of the current is opposite the direction of flow of electrons. For a beam of positively charged protons in an accelerator, however, the current is in the direction of motion of the protons. In some cases – such as those involving gases and electrolytes, for instance – the current is the result of the flow of both positive and negative charges. It is common to refer to a moving charge (positive or negative) as a mobile **charge carrier.**

If the ends of a conducting wire are connected to form a loop, all points on the loop are at the same electric potential; hence, the electric field is zero within and at the surface of the conductor. Because the electric field is zero, there is no net transport of charge through the wire; therefore, there is no current.

If the ends of the conducting wire are connected to a battery, however, all points on the loop are not at the same potential. The battery sets up a potential difference between the ends of the loop, creating an electric field within the wire. The electric field exerts forces on the electrons in the wire, causing them to move in the wire and therefore creating a current.

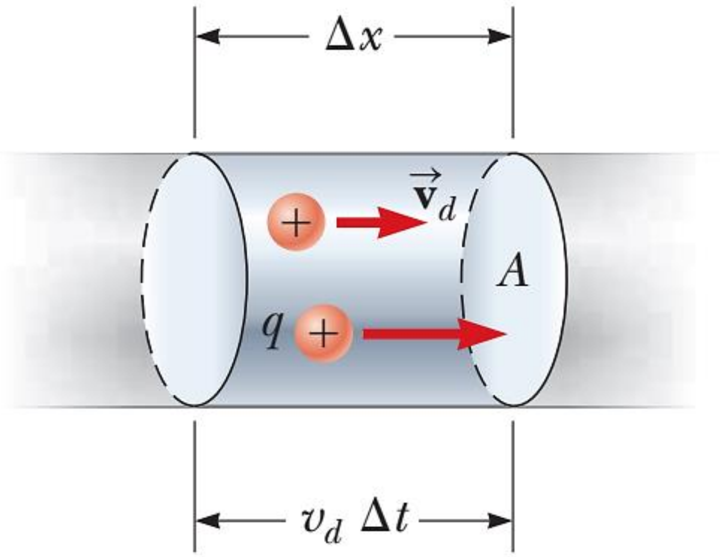
Microscopic model of current

We can relate current to the motion of the charge carriers by describing a microscopic model of conduction in a metal. Consider the current in a cylindrical conductor of cross-sectional area A (see the Figure). The volume of a segment of the conductor of length Δx (between the two circular cross sections shown in Fig.) is $A\Delta$x. If n represents the number of mobile charge carriers per unit volume (in other words, the charge carrier density), the number of carriers in the segment is $nA\Delta x$.

Therefore, the total charge ΔQ in this segment is

$$\Delta Q = (nA\Delta x)q$$

where q is the charge on each carrier. If the carriers move with a velocity $\mathbf{v}_d$ parallel to the axis of the cylinder, the magnitude of the displacement they experience in the x direction in a time interval Δt is $\Delta x = v_d \Delta t$. Let Δt be the time interval required for the charge carriers in the segment to move through a displacement whose magnitude is equal to the length of the segment.



A segment of a uniform conductor of cross-sectional area A.

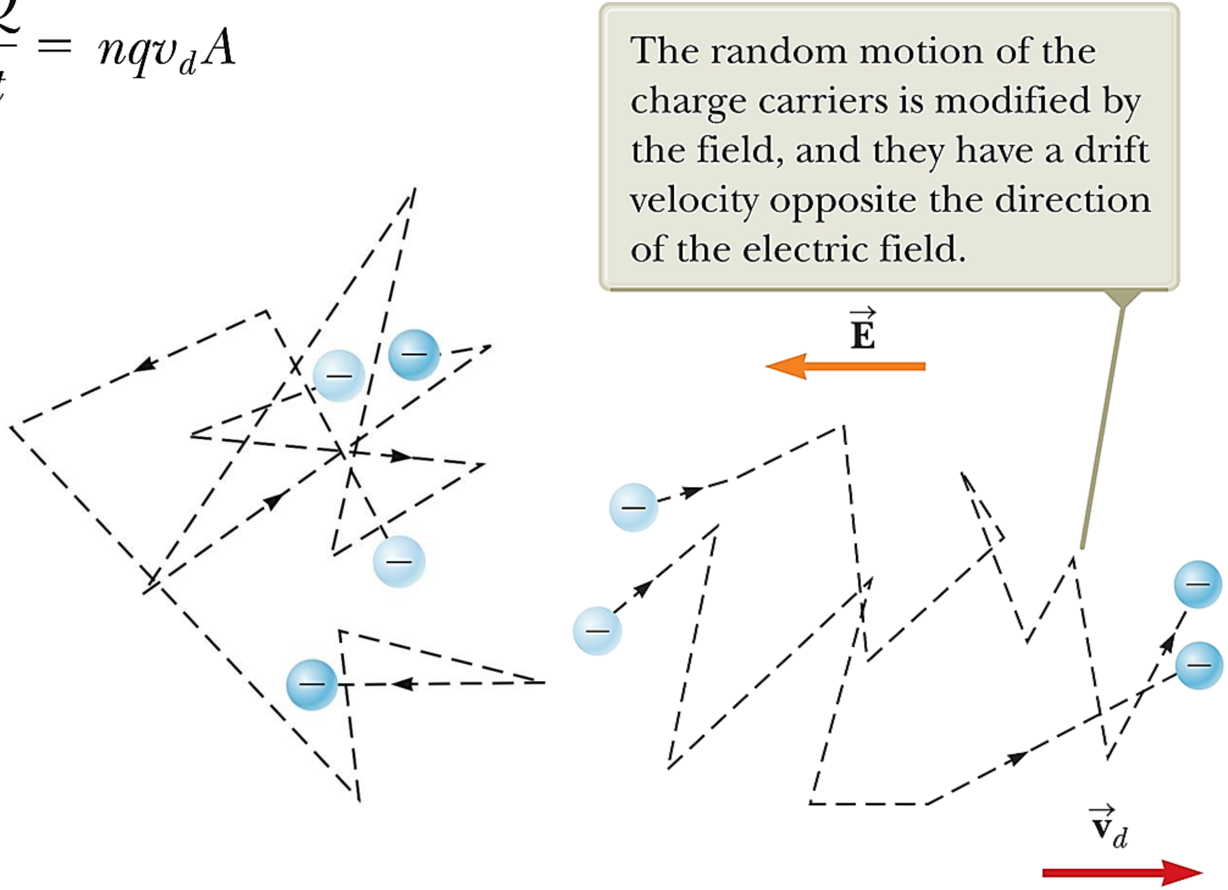
This time interval is also the same as that required for all the charge carriers in the segment to pass through the circular area at one end. With this choice, we can write ΔQ as

$$\Delta Q = (nAv_d\,\Delta t)q$$

Dividing both sides of this equation by Δt, we find that the average current in the conductor is

$$I_{avg} = \frac{\Delta Q}{\Delta t} = nqv_dA$$

In reality, the speed of the charge carriers v_d is an average speed called the **drift speed.** To understand the meaning of drift speed, consider a conductor in which the charge carriers are free electrons. If the conductor is isolated – that is, the potential difference across it is zero – these electrons undergo random motion that is analogous to the motion of gas molecules. The electrons collide repeatedly with the metal atoms, and their resultant motion is complicated and zigzagged as in Figure a.



a

b

As discussed earlier, when a potential difference is applied across the conductor (for example, by means of a battery), an electric field is set up in the conductor; this field exerts an electric force on the electrons, producing a current. In addition to the zigzag motion due to the collisions with the metal atoms, the electrons move slowly along the conductor (in a direction opposite that of $\boldsymbol{E}$) at the **drift velocity** $\boldsymbol{v}_d$ as shown in Figure b.

You can think of the atom–electron collisions in a conductor as an effective internal friction (or drag force) similar to that experienced by a liquid's molecules flowing through a pipe stuffed with steel wool. The energy transferred from the electrons to the metal atoms during collisions causes an increase in the atom's vibrational energy and a corresponding increase in the conductor's temperature.

Resistance

Consider a conductor of cross-sectional area A carrying a current I. The **current density** J in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

$$J \equiv \frac{I}{A} = nqv_d$$

where J has SI units of amperes per meter squared. This expression is valid only if the current density is uniform and only if the surface of cross-sectional area A is perpendicular to the direction of the current. A current density and an electric field are established in a conductor whenever a potential difference is maintained across the conductor. In some materials, the current density is proportional to the electric field:

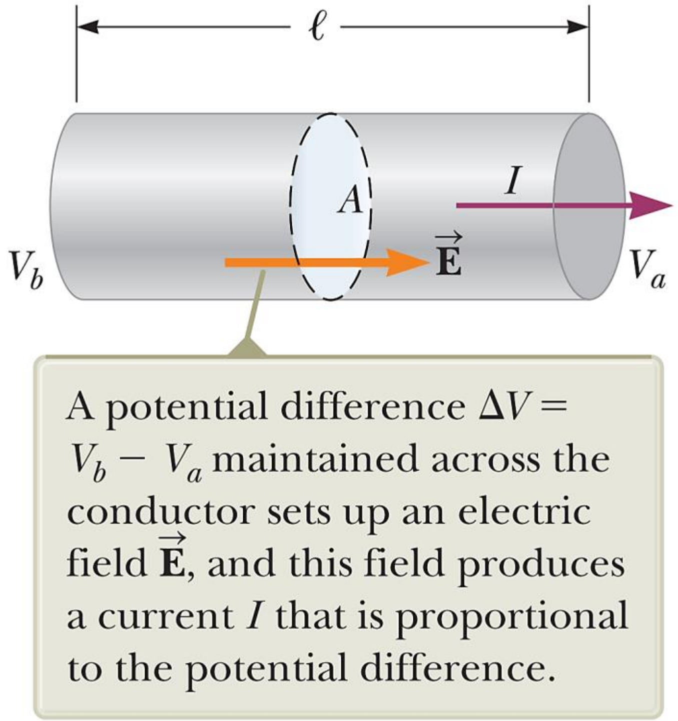
$$J = \sigma E$$

where the constant of proportionality σ is called the **conductivity** of the conductor. Do not confuse conductivity σ with surface charge density, for which the same symbol is used. Materials that obey the Equation are said to follow **Ohm's law,** named after Georg Simon Ohm. More specifically, Ohm's law states the following:

> For many materials (including most metals), the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current.

Materials and devices that obey Ohm's law and hence demonstrate this simple relationship between E and J are said to be *ohmic*. Experimentally, however, it is found that not all materials and devices have this property. Those that do not obey Ohm's law are said to be *nonohmic*. Ohm's law is not a fundamental law of nature; rather, it is an empirical relationship valid only for certain situations. We can obtain an equation useful in practical applications by considering a segment of straight wire of uniform cross-sectional area A and length ℓ, as shown in the Figure. A potential difference $\Delta V = V_b - V_a$ is maintained across the wire, creating in the wire an electric field and a current. If the field is assumed to be uniform, the magnitude of the potential difference across the wire is related to the field within the wire, as we said in the previous lecture,

$$\Delta V = E\ell,$$



A uniform conductor of length ℓ and cross-sectional area A.

Therefore, we can express the current density in the wire as

$$J = \sigma \frac{\Delta V}{\ell}$$

Because $J = I/A$, the potential difference across the wire is

$$\Delta V = \frac{\ell}{\sigma} J = \left(\frac{\ell}{\sigma A}\right) I = R\, I$$

The quantity $R = \ell/\sigma A$ is called the **resistance** of the conductor. We define the resistance as the ratio of the potential difference across a conductor to the current in the conductor:

$$R \equiv \frac{\Delta V}{I}$$

Many individuals call the last Equation Ohm's law, but that is incorrect. This equation is simply the definition of resistance, and it provides an important relationship between voltage, current, and resistance. Ohm's law is related to a proportionality of J to E or, equivalently, of I to ΔV, which, from the last Equation, indicates that the resistance is constant, independent of the applied voltage. We can see some devices for which the last Equation correctly describes their resistance, but that do *not* obey Ohm's law.

This result shows that resistance has SI units of volts per ampere. One volt per ampere is defined to be one **ohm** (Ω):

$$1\ \Omega \equiv 1\ \mathrm{V/A}$$

The last Equation shows that if a potential difference of 1 V across a conductor causes a current of 1 A, the resistance of the conductor is 1 Ω. For example, if an electrical appliance connected to a 120-V source of potential difference carries a current of 6 A, its resistance is 20 Ω.

Most electric circuits use circuit elements called **resistors** to control the current in the various parts of the circuit. The many resistors are built into integrated circuit chips, but stand-alone resistors are still available and widely used. Two common types are the *composition resistor,* which contains carbon, and the *wire-wound resistor,* which consists of a coil of wire.

The inverse of conductivity is **resistivity** ρ (do not confuse resistivity ρ with mass density or charge density, for which the same symbol is used):

$$\rho = \frac{1}{\sigma}$$

where ρ has the units ohm · meters (Ω · m). Because $R = \ell/\sigma A$, we can express the resistance of a uniform block of material along the length ℓ, as

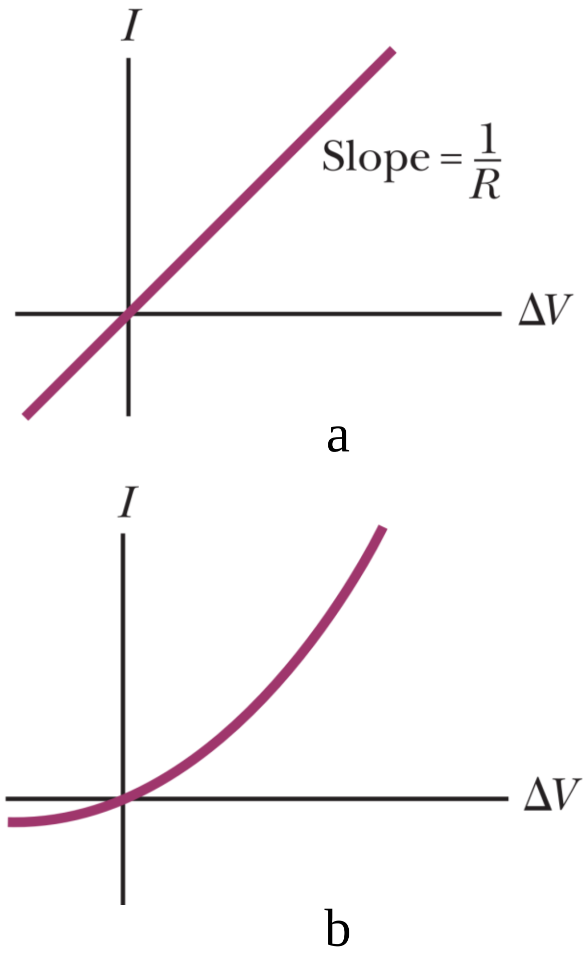
$$\boxed{R = \rho \frac{\ell}{A}}$$

Resistivity is a property of a *substance*, whereas resistance is a property of an *object*. We have seen similar pairs of variables before. For example, density is a property of a substance, whereas mass is a property of an object.

Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature. In addition, as you can see from the last Equation, the resistance of a sample of the material depends on the geometry of the sample as well as on the resistivity of the material. The next Table gives the resistivities of a variety of materials at 20°C. Notice the enormous range, from very low values for good conductors such as copper and silver to very high values for good insulators such as glass and rubber. An ideal conductor would have zero resistivity, and an ideal insulator would have infinite resistivity.

The next Equation shows that the resistance of a given cylindrical conductor such as a wire is proportional to its length and inversely proportional to its cross-sectional area. If the length of a wire is doubled, its resistance doubles. If its cross-sectional area is doubled, its resistance decreases by one half. The situation is analogous to the flow of a liquid through a pipe. As the pipe's length is increased, the resistance to flow increases. As the pipe's cross-sectional area is increased, more liquid crosses a given cross section of the pipe per unit time interval. Therefore, more liquid flows for the same pressure differential applied to the pipe, and the resistance to flow decreases.

Ohmic materials and devices have a linear current–potential difference relationship over a broad range of applied potential differences (see the Figure a). The slope of the I-versus-ΔV curve in the linear region yields a value for $1/R$. Nonohmic materials have a nonlinear current–potential difference relationship. One common semiconducting device with nonlinear I-versus-ΔV characteristics is the *junction diode* (Fig. b). The resistance of this device is low for currents in one direction (positive ΔV) and high for currents in the reverse direction (negative ΔV). In fact, most modern electronic devices, such as transistors, have nonlinear current–potential difference relationships; their proper operation depends on the particular way they violate Ohm's law.



(a) The current–potential difference curve for an ohmic material. The curve is linear, and the slope is equal to the inverse of the resistance of the conductor. (b) A nonlinear current–potential difference curve for a junction diode. This device does not obey Ohm's law.

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] α [$(°C)^{-1}$]
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.00×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon[d]	2.3×10^{3}	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C. All elements in this table are assumed to be free of impurities.

[c] A nickel–chromium alloy commonly used in heating elements. The resistivity of Nichrome varies with composition and ranges between 1.00×10^{-6} and 1.50×10^{-6} $\Omega \cdot m$.

[d] The resistivity of silicon is very sensitive to purity. The value can be changed by several orders of magnitude when it is doped with other atoms.

Resistance and Temperature

Over a limited temperature range, the resistivity of a conductor varies approximately linearly with temperature according to the expression

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

where ρ is the resistivity at some temperature T (in Kelvins or degrees Celsius), ρ_0 is the resistivity at some reference temperature T_0 (usually taken to be 20°C), and α is the **temperature coefficient of resistivity.** From the Equation, the temperature coefficient of resistivity can be expressed as

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T}$$

where $\Delta\rho = \rho - \rho_0$ is the change in resistivity in the temperature interval $\Delta T = T - T_0$.

The temperature coefficients of resistivity for various materials are given in the previous Table. Notice that the unit for α is degrees Celsius^{-1} [$(°C)^{-1}$]. Because resistance is proportional to resistivity (see above), the variation of resistance of a sample is

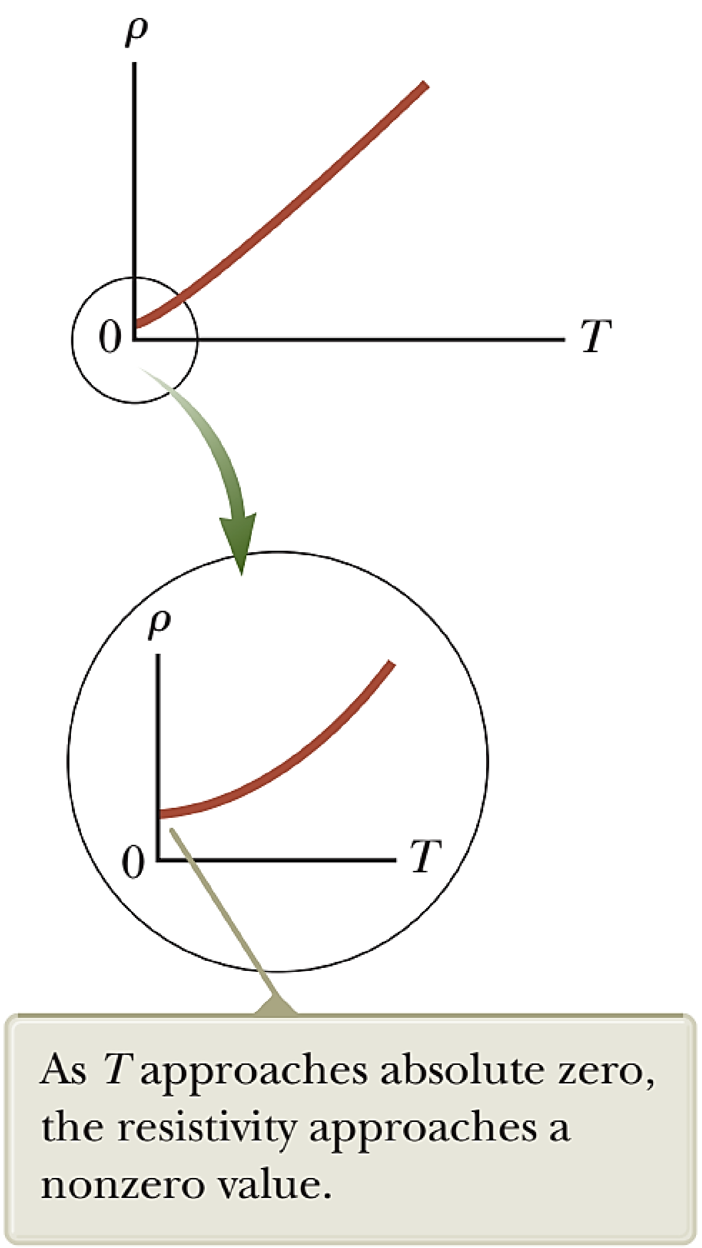
$$R = R_0[1 + \alpha(T - T_0)]$$

where R_0 is the resistance at temperature T_0. Use of this property enables precise temperature measurements through careful monitoring of the resistance of a probe made from a particular material.

For some metals such as copper, resistivity is nearly proportional to temperature as shown in the Figure. A nonlinear region always exists at very low temperatures, however, and the resistivity usually reaches some finite value as the temperature approaches absolute zero.

This residual resistivity near absolute zero is caused primarily by the collision of electrons with impurities and imperfections in the metal. In contrast, high-temperature resistivity (the linear region) is predominantly characterized by collisions between electrons and metal atoms.

Notice that three of the α values in the Table above are negative, indicating that the resistivity of these materials decreases with increasing temperature. This behavior is indicative of a class of materials called *semiconductors*, and is due to an increase in the density of charge carriers at higher temperatures. Because the charge carriers in a semiconductor are often associated with impurity atoms, the resistivity of these materials is very sensitive to the type and concentration of such impurities.

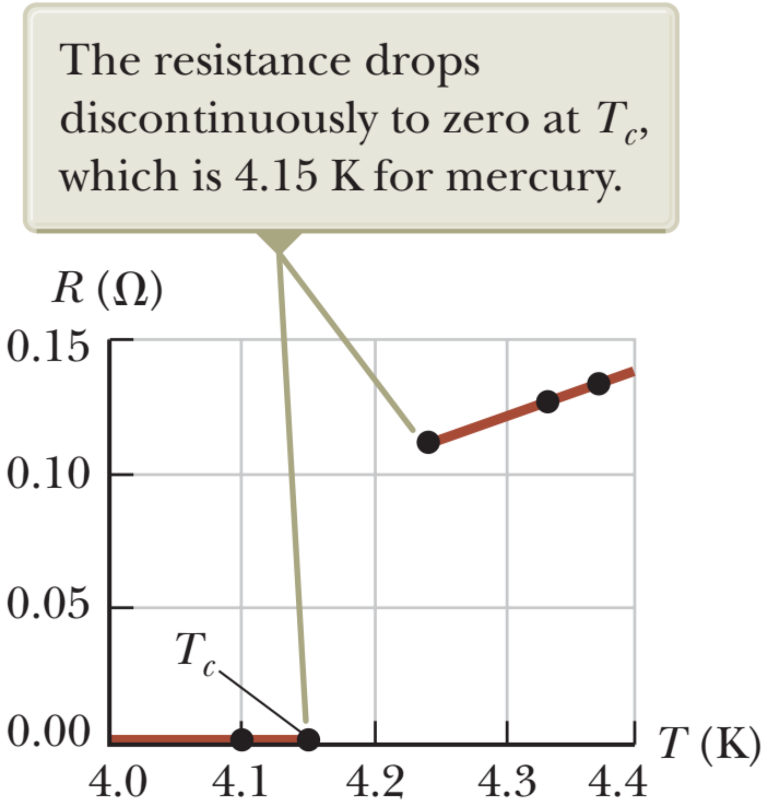


Resistivity versus temperature for a metal such as copper. The curve is linear over a wide range of temperatures, and ρ increases with increasing temperature.

Superconductors

There is a class of metals and compounds whose resistance decreases to zero when they are below a certain temperature T_c, known as the **critical temperature.** These materials are known as **superconductors.** The resistance–temperature graph for a superconductor follows that of a normal metal at temperatures above T_c (see the Figure). When the temperature is at or below T_c, the resistivity drops suddenly to zero. This phenomenon was discovered in 1911 by Dutch physicist Heike Kamerlingh-Onnes (1853–1926) as he worked with mercury, which is a superconductor below 4.2 K. Measurements have shown that the resistivities of superconductors below their T_c values are less than $4 \times 10^{-25}\ \Omega \cdot \mathrm{m}$, or approximately 10^{17} times smaller than the resistivity of copper. In practice, these resistivities are considered to be zero.

Today, thousands of superconductors are known, and as the next Table illustrates, the critical temperatures of recently discovered superconductors are substantially higher than initially thought possible. Two kinds of superconductors are recognized. The more recently identified ones are essentially ceramics with high critical temperatures, whereas superconducting materials such as those observed by Kamerlingh-Onnes are metals.



Resistance versus temperature for a sample of mercury (Hg). The graph follows that of a normal metal above the critical temperature T_c.

A small permanent magnet levitated above a disk of the superconductor $YBa_2Cu_3O_7$, which is in liquid nitrogen at 77 K.

Critical Temperatures for Various Superconductors

Material	T_c (K)
$HgBa_2Ca_2Cu_3O_8$	134
Tl—Ba—Ca—Cu—O	125
Bi—Sr—Ca—Cu—O	105
$YBa_2Cu_3O_7$	92
Nb_3Ge	23.2
Nb_3Sn	18.05
Nb	9.46
Pb	7.18
Hg	4.15
Sn	3.72
Al	1.19
Zn	0.88

If a room-temperature superconductor is ever identified, its effect on technology could be tremendous.

The value of T_c is sensitive to chemical composition, pressure, and molecular structure. Copper, silver, and gold, which are excellent conductors, do not exhibit superconductivity.

One truly remarkable feature of superconductors is that once a current is set up in them, it persists *without any applied potential difference* (because R = 0). Steady currents have been observed to persist in superconducting loops for several years with no apparent decay!

An important and useful application of superconductivity is in the development of superconducting magnets, in which the magnitudes of the magnetic field are approximately ten times greater than those produced by the best normal electromagnets. Such superconducting magnets are being considered as a means of storing energy. Superconducting magnets are currently used in medical magnetic resonance imaging, or MRI, units, which produce high-quality images of internal organs without the need for excessive exposure of patients to X-rays or other harmful radiation.

Resistance versus temperature for a sample of mercury (Hg). The graph follows that of a normal metal above the critical temperature T_c.

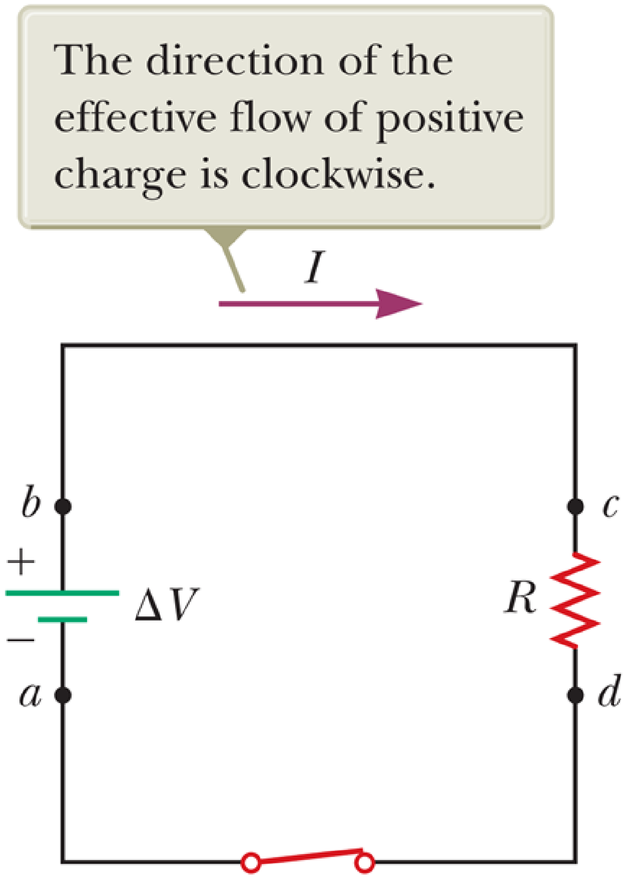
Electrical power

In typical electric circuits, energy T_{ET} is transferred by electrical transmission from a source such as a battery to some device such as a lightbulb or a radio receiver. Let's determine an expression that will allow us to calculate the rate of this energy transfer. First, consider the simple circuit in the Figure, where energy is delivered to a resistor. (Resistors are designated by the circuit zigzag symbol.) Because the connecting wires also have resistance, some energy is delivered to the wires and some to the resistor. Unless noted otherwise, we shall assume the resistance of the wires is small compared with the resistance of the circuit element so that the energy delivered to the wires is negligible.

Imagine following a positive quantity of charge Q moving clockwise around the circuit in the Figure from point a through the battery and resistor back to point a. We identify the entire circuit as our system. As the charge moves from a to b through the battery, the electric potential energy of the system *increases* by an amount $Q\Delta V$ while the chemical potential energy in the battery *decreases* by the same amount.

(Recall that $\Delta U = q\Delta V$.) As the charge moves from c to d through the resistor, however, the electric potential energy of the system decreases due to collisions of electrons with atoms in the resistor. In this process, the electric potential energy is transformed to internal energy corresponding to increased vibrational motion of the atoms in the resistor. Because the resistance of the interconnecting wires is neglected, no energy transformation occurs for paths *bc* and *da*. When the charge returns to point a, the net result is that some of the chemical potential energy in the battery has been delivered to the resistor and resides in the resistor as internal energy E_{int} associated with molecular vibration.

The resistor is normally in contact with air, so its increased temperature results in a transfer of energy by heat Q into the air. In addition, the resistor emits thermal radiation T_{ER}, representing another means of escape for the energy. After some time interval has passed, the resistor reaches a constant temperature. At this time, the input of energy from the battery is balanced by the output of energy from the resistor by heat and radiation, and the resistor is a nonisolated system in steady state. Some electrical devices include *heat sinks* connected to parts of the circuit to prevent these parts from reaching dangerously high temperatures. Heat sinks are pieces of metal with many fins.



A circuit consisting of a resistor of resistance R and a battery having a potential difference ΔV across its terminals.

Because the metal's high thermal conductivity provides a rapid transfer of energy by heat away from the hot component and the large number of fins provides a large surface area in contact with the air, energy can transfer by radiation and into the air by heat at a high rate.

Let's now investigate the rate at which the electric potential energy of the system decreases as the charge Q passes through the resistor:

$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

where I is the current in the circuit. The system regains this potential energy when the charge passes through the battery, at the expense of chemical energy in the battery. The rate at which the potential energy of the system decreases as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor. Therefore, the power P, representing the rate at which energy is delivered to the resistor, is

$$P = I\Delta V$$

We derived this result by considering a battery delivering energy to a resistor. This Equation, however, can be used to calculate the power delivered by a voltage source to *any* device carrying a current I and having a potential difference ΔV between its terminals.

Using the Equation and $\Delta V = IR$ for a resistor, we can express the power delivered to the resistor in the alternative forms

$$P = I^2R = \frac{(\Delta V)^2}{R}$$

When I is expressed in amperes, ΔV in volts, and R in ohms, the SI unit of power is the watt, as it was earlier in our discussion of mechanical power. The process by which energy is transformed to internal energy in a conductor of resistance R is often called *joule heating;* this transformation is also often referred to as an I^2R loss.

When transporting energy by electricity through power lines, you should not assume the lines have zero resistance. Real power lines do indeed have resistance, and power is delivered to the resistance of these wires. Utility companies seek to minimize the energy transformed to internal energy in the lines and maximize the energy delivered to the consumer. Because $P = I\Delta V$, the same amount of energy can be transported either at high currents and low potential differences or at low currents and high potential differences. Utility companies choose to transport energy at low currents and high potential differences primarily for economic reasons. Copper wire is very expensive, so it is cheaper to use high-resistance wire (that is, wire having a small cross-sectional area; see the corresponding Equation above). Therefore, in the expression for the power delivered to a resistor, $P = I^2R$, the resistance of the wire is fixed at a relatively high value for economic considerations. The I^2R loss can be reduced by keeping the current I as low as possible, which means transferring the energy at a high voltage. In some instances, power is transported at potential differences as great as 765 kV. At the destination of the energy, the potential difference is usually reduced to 4 kV by a device called a *transformer.* Another transformer drops the potential difference to 240 V for use in your home. Of course, each time the potential difference decreases, the current increases by the same factor and the power remains the same.